Lecture Notes

on

Electric Power II- (Second Semester)

(EE3318)

By

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2023-2024

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References:

- Element of Power System Analysis / by W. Stevenson, McGraw- Hill Pub., 2005.
- Principles of power system / by V.K Mehta and S. chand, company ltd., 2004.

Chapter One

Characteristics of Transmission Line and Its Power flow

1.1- Introduction to Two-port Networks

1.2- Determination of Generalized Constants for Transmission Lines

1.3- Two Transmission Lines in Cascaded (Series)

1.4- Two Transmission Lines in Parallel

1.5- Power Flow through a Transmission Line

1.6- Examples

1.1- Introduction to Two-port Networks:

- A pair of terminals through which a current may enter or leave a network is known as a port.
- Two-terminal devices or elements such as resistors, capacitors, and inductors) result in one-port networks. Most of the circuits we have dealt with so far are two-terminal or one-port circuits, represented in figure 1.1.

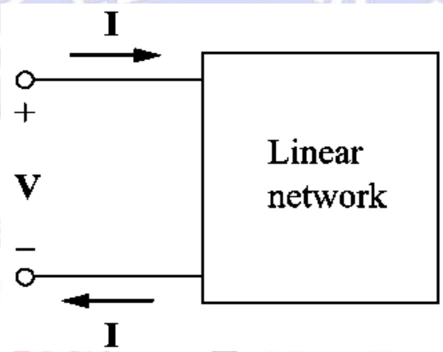


Fig. 1.1:

- We have also studied four-terminal or two-port circuits involving op amps, transistors, and transformers, as shown in fig. 1.2.

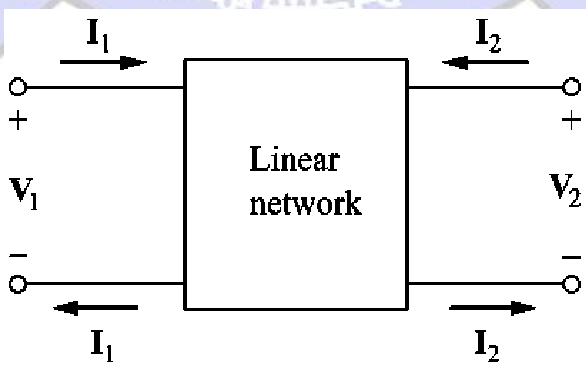


Fig. 1.2:

- A two-port network is an electrical network with two separate ports for input (V_1 and I_1) and output (V_2 and I_2).
- The two-port network is useful to be used in different fields such as communications, control systems, power systems, and electronics.
- There different types of two-port network such as impedance parameters, admittance parameters, hybrid parameters, and transmission parameters (ABCD).

1.2- Determination of Generalized Constants for Transmission Lines:

- In any four-terminal network, the input voltage and input current can be expressed in terms of output voltage and output current.
- Incidentally, a transmission line is a 4-terminal network; two input terminals where power enters the network and two output terminals where power leaves the network.
- The transmission line can be represented as tow-port network as shown in figure 1.3.

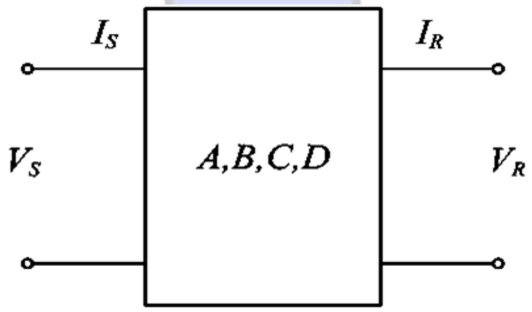


Fig. 1.3:

Therefore, the input voltage $(\vec{V_S})$ and input current $(\vec{I_S})$ of a 3-phase transmission line can be expressed as :

$$\vec{V_S} = \vec{A}\,\vec{V_R} + \vec{B}\,\vec{I_R}$$
$$\vec{I_S} = \vec{C}\,\vec{V_R} + \vec{D}\,\vec{I_R}$$

where
$\vec{V_S}$ = sending end voltage per phase
$\vec{I_S}$ = sending end current
$\vec{V_R}$ = receiving end voltage per phase
$\vec{I_R}$ = receiving end current

The following points may be kept in mind :

(i) The constants $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ are generally complex numbers.

(ii) The constants $\vec{A}$ and $\vec{D}$ are dimensionless whereas the dimensions of $\vec{B}$ and $\vec{C}$ are ohms and siemen respectively.

(iii) For a given transmisson line,

$$\vec{A} = \vec{D}$$

- The derivation of the A, B, C, D parameters for different types of transmission lines is as listed below:

1.2.1- ABCD Constants for Short Transmission Lines:

- In short transmission lines, the effect of line capacitance is neglected. Therefore, the line is considered to have series impedance.
- Figure 1.4 shows the short transmission line circuit of a 3-phase transmission line on a single-phase basis.

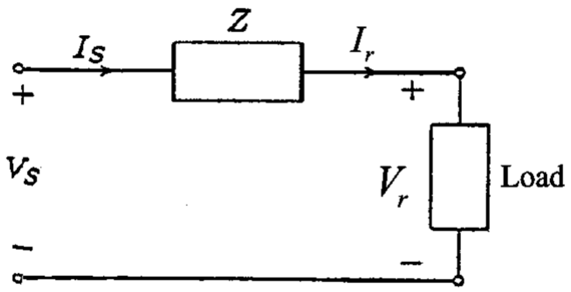


Fig. 1.4:

- The circuit is solved as a simple series A.C circuit. So,

$$\vec{I}_S = \vec{I}_r$$
$$\vec{V}_S = \vec{V}_r + Z\vec{I}_r$$

- The above equations can be written in matrix form as:

$$\vec{V}_S = \vec{A}\,\vec{V}_R + \vec{B}\,\vec{I}_R$$
$$\vec{I}_S = \vec{C}\,\vec{V}_R + \vec{D}\,\vec{I}_R$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

Comparing with eqs., we have

$\vec{A} = 1$; $\vec{B} = \vec{Z}$, $\vec{C} = 0$ and $\vec{D} = 1$

- Voltage regulation of a transmission line is defined as the rise in voltage at the receiving-end,

$$\text{Per cent regulation} = \frac{|V_{R0}| - |V_{RL}|}{|V_{RL}|} \times 100$$

where

$|V_{R0}|$ = magnitude of no load receiving-end voltage

$|V_{RL}|$ = magnitude of full load receiving-end voltage (at a specified power factor)

For short line, $|V_{R0}| = |V_S|$, $|V_{RL}| = |V_R|$

$$\therefore \quad \text{Per cent regulation} = \frac{|V_S| - |V_R|}{|V_R|}$$

Example 1.1:

A single-phase 50 Hz generator supplies an inductive load of 5,000 kW at a power factor of 0.707 lagging by means of an overhead transmission line 20 km long. The line resistance and inductance are 0.0195 ohm and 0.63 mH per km. The voltage at the receiving-end is required to be kept constant at 10 kV.

Find (a) the sending-end voltage,
(b) the voltage regulation of the line
(c) the transmission efficiency

Given:-

single-phase, 50 Hz, $P_r = 5000$ kW

p.f = 0.707, $l = 20$ km, $V_r = 10$ KV

$R = 0.0195\ \Omega$ & $L = 0.63$ mH

Sol:-

Total Resistance $= R = 0.0195 \times 20 = 0.39\ \Omega$

Total Reactance $= X_L = 2\pi \times 50 \times 0.63 \times 10^{-3} \times 20$
$= 3.96\ \Omega$

$\therefore Z = R + jX_L = 0.39 + j3.96\ \Omega$

$\therefore I = I_s = I_r = \frac{P_r}{V_r * p.f} = \frac{5000}{10 * 0.707}$

$\Rightarrow I_r = 707.2\ A$; Lag $\Rightarrow \theta = \cos^{-1}(0.707)$

$\therefore I_r = 707.2 \angle -45^\circ$

@

To Find Vs $\Rightarrow V_s = A V_r + B I_r$

Where $A = 1$ & $B = Z$

$\therefore V_s = 10\angle 0 * 10^3 + (0.39 + j3.96) * 707.2 \angle -45$

$\Rightarrow V_s = 12.31 \angle 8.34$ KV

1.2.2- ABCD Constants for Medium Transmission Lines:

- To derive the A, B, C, D parameters of **nominal π method** for medium transmission line as shown in figure 1.5.

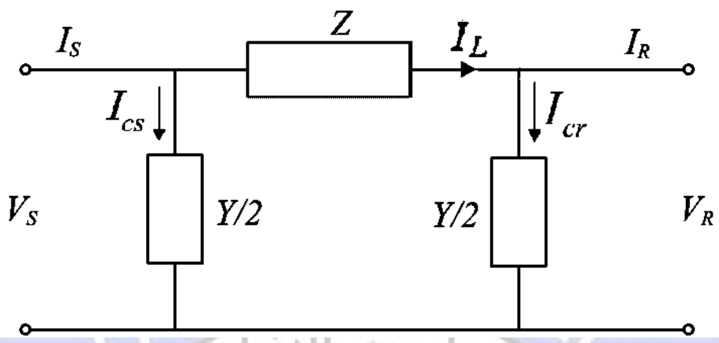


Fig. 1.5:

- **In the nominal π circuit**, the shunt admittance is divided into two halves; one half with value $(Y/2)$ is concentrated at the load end and the other $(Y/2)$ half is at the sending end. While the total series impedance is placed at the middle of circuit.

- Hence, by using KCL and KVL, we can write the circuit equation as follows:

$$I_L = I_r + I_{cr}$$

$$= I_r + V_r \frac{Y}{2}$$

$$V_S = V_r + I_L Z$$
$$= V_r + \left(I_r + V_r \frac{Y}{2}\right) Z$$
$$= V_r + I_r Z + V_r \frac{YZ}{2}$$
$$= V_r \left(1 + \frac{YZ}{2}\right) + I_r Z \quad \ldots\ldots\ldots\ldots(3)$$

$$I_S = I_{cs} + I_L = I_{cs} + I_{cr} + I_r$$
$$= V_S \frac{Y}{2} + V_r \frac{Y}{2} + I_r$$
$$= \left[V_r\left(1 + \frac{YZ}{2}\right) + I_r Z\right] \times \frac{Y}{2} + V_r \frac{Y}{2} + I_r$$
$$= V_r\left(Y + \frac{Y^2 Z}{4}\right) + I_r\left(1 + \frac{YZ}{2}\right)$$
$$= V_r Y\left(1 + \frac{YZ}{4}\right) + I_r\left(1 + \frac{YZ}{2}\right) \quad \ldots\ldots\ldots\ldots(4)$$

Eq. (3) and (4) can be written in the matrix form as:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{YZ}{2}\right) & Z \\ Y\left(1 + \frac{YZ}{4}\right) & \left(1 + \frac{YZ}{2}\right) \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots(5)$$

- By comparing above equation with the standard ABCD parameter equation;

$$V_s = A\,V_r + B\,I_r$$
$$I_s = C\,V_r + D\,I_r$$

- We get for medium transmission line (**nominal π method**);

$$\vec{A} = \left(1 + \frac{\vec{Y}\vec{Z}}{2}\right) \qquad \vec{B} = \vec{Z}$$

$$\vec{C} = \vec{Y}\left(1 + \frac{\vec{Y}\vec{Z}}{4}\right) \qquad \vec{D} = \left(1 + \frac{\vec{Y}\vec{Z}}{2}\right)$$

- To derive the A, B, C, D parameters of **nominal T method** for medium transmission line as shown in figure 1.6.

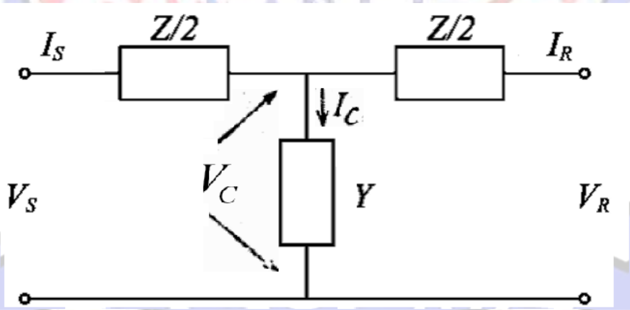


Fig. 1.6:

Where;

'I_c'- Current flowing in the shunt branch of the circuit

'V_c'- Voltage at the node of shunt branch of the circuit

- **In the nominal T circuit**, the total shunt admittance of each conductor is concentrated at the center of the line, while the series impedance is split into two equal parts ($Z/2$ *in each side*).

- Hence, by using KCL and KVL, we can write the circuit equation as follows:

$$I_s = I_r + I_c = I_r + V_c Y$$

$$= I_r + \left(V_r + I_r \frac{Z}{2}\right) Y$$

$$= I_r \left(1 + \frac{YZ}{2}\right) + V_r Y \qquad \text{.......... ....}(6)$$

$$V_S = V_r + I_r \frac{Z}{2} + I_s \frac{Z}{2}$$

$$= V_r + I_r \frac{Z}{2} + \left[I_r \left(1 + \frac{YZ}{2}\right) + V_r Y \right] \times \frac{Z}{2}$$

$$= V_r \left(1 + \frac{YZ}{2}\right) + I_r Z \left(1 + \frac{YZ}{4}\right) \qquad \text{.......... ....}(7)$$

Eq. (6) and (7) can be written in the matrix form as :

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{YZ}{2}\right) & Z\left(1 + \frac{YZ}{4}\right) \\ Y & \left(1 + \frac{YZ}{2}\right) \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix} \qquad \text{...............}(8)$$

- By comparing above equation with the standard ABCD parameter equation;

$$V_s = A V_r + B I_r$$
$$I_s = C V_r + D I_r$$

- We get for medium transmission line (**nominal T method**);

$$\vec{A} = 1 + \frac{\vec{Y}\vec{Z}}{2} \qquad \vec{B} = \vec{Z}\left(1 + \frac{\vec{Y}\vec{Z}}{4}\right)$$

$$\vec{C} = \vec{Y} \qquad \vec{D} = 1 + \frac{\vec{Y}\vec{Z}}{2}$$

- To find voltage regulation for medium transmission line, by letting I_R is zero ($I_R = 0$) for the following equation;

Now, $\vec{V_S} = \vec{A}\,\vec{V_R} + \vec{B}\,\vec{I_R}$

At no load, $\vec{I_R} = 0$

$\therefore \quad \vec{V_S} = \vec{A}\,\vec{V_{R0}}$

- We see that the A is the ratio V_s/V_R ($A = V_s/V_{R0}$) at no load. The constant A is useful in computing voltage regulation.
- In general, the equation of voltage regulation is:

$$\text{Per cent regulation} = \frac{|V_{R0}| - |V_{RL}|}{|V_{RL}|} \times 100$$

where $\overrightarrow{V_{R0}}$ = voltage at receiving end at no load

or $\overrightarrow{V_{R0}} = \vec{V_S}/\vec{A}$

or $V_{R0} = V_S/A$ (in magnitude)

$$\therefore \quad \% \text{ Regulation} = \frac{(V_S/A - V_R)}{V_R} \times 100$$

Example 1.2:

A balanced 3-phase load of 30 MW is supplied at 132 kV, 50 Hz and 0·85 p.f. lagging by means of a transmission line. The series impedance of a single conductor is (20 + j52) ohms and the total phase-neutral admittance is 315×10^{-6} siemen. Using nominal T method, determine: (i) the A, B, C and D constants of the line (ii) sending end voltage (iii) regulation of the line.

Solution.

Series line impedance/phase, $\vec{Z} = (20 + j\,52)\ \Omega$

Shunt admittance/phase, $\vec{Y} = j\,315 \times 10^{-6}$ S

(*i*) **Generalised constants of line.** For nominal *T* method

$$\vec{A} = \vec{D} = 1 + \vec{Z}\,\vec{Y}/2$$

$$= 1 + \frac{20 + j\,52}{2} \times j\,315 \times 10^{-6}$$

$$= 0{\cdot}992 + j\,0{\cdot}00315 = \mathbf{0{\cdot}992 \angle 0{\cdot}18^\circ}$$

$$\vec{B} = \vec{Z}\left(1 + \frac{\vec{Z}\,\vec{Y}}{4}\right)$$

$$= (20 + j\,52)\left[1 + \frac{(20 + j\,52)j\,315 \times 10^{-6}}{4}\right]$$

$$= 19{\cdot}84 + j\,51{\cdot}82 = \mathbf{55{\cdot}5\ \angle 69^{o}}$$

$$\vec{C} = \vec{Y} = \mathbf{0{\cdot}000315\ \ \angle 90^{o}}$$

(*ii*) Sending end voltage.

Receiving end voltage/phase,

$$V_R = 132 \times 10^3/\sqrt{3} = 76210\text{ V}$$

Receiving end current,

$$I_R = \frac{30 \times 10^6}{\sqrt{3} \times 132 \times 10^3 \times 0{\cdot}85} = 154\text{ A}$$

$$\cos\phi_R = 0{\cdot}85\ ; \quad \sin\phi_R = 0{\cdot}53$$

Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j0 = 76210\text{ V}$$

$$\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$$

$$= 154\,(0{\cdot}85 - j\,0{\cdot}53) = 131 - j\,81{\cdot}62$$

Sending end voltage per phase is

$$\vec{V_S} = \vec{A}\ \vec{V_R} + \vec{B}\ \vec{I_R}$$

$$= (0{\cdot}992 + j\,0{\cdot}0032)\,76210 + (19{\cdot}84 + j\,51{\cdot}82)\,(131 - j\,81{\cdot}62)$$

$$= 82{,}428 + j\,5413$$

∴ Magnitude of sending end voltage is

$$V_S = \sqrt{(82{,}428)^2 + (5413)^2}$$

$$= 82{\cdot}6 \times 10^3 \text{V} = 82{\cdot}6 \text{ kV}$$

∴ Sending end line-to-line voltage

$$= 82{\cdot}6 \times \sqrt{3} = \mathbf{143\ kV}$$

(*iii*) Regulation.

$$\% \text{ Regulation} = \frac{(V_S/A - V_R)}{V_R} \times 100$$

$$= \frac{(82 \cdot 6/0 \cdot 992) - 76 \cdot 21}{76 \cdot 21} \times 100$$

$$= \mathbf{9{\cdot}25\%}$$

1.2.3- ABCD Constants for Long Transmission Lines:

- The general relationships of long transmission line for the voltage and current in both sending-end and receiving-end sides are:

$$V_S = (\cosh \gamma l)\, V_r + Z_c (\sinh \gamma l)\, I_r$$

$$I_S = \frac{1}{Z_c}(\sinh \gamma l)\, V_r + (\cosh \gamma l)\, I_r$$

- By comparing the above equations with the following standard equation of ABCD parameters:

$$V_s = A\,V_r + B\,I_r$$

$$I_s = C\,V_r + D\,I_r$$

- We get for the long transmission line;

$$A = \cosh(\gamma l) \quad ; \quad B = Z_C \sinh(\gamma l)$$

$$C = \frac{1}{Z_C} \sinh(\gamma l) \;;\quad D = \cosh(\gamma l\,)$$

- Comparison of ABCD parameters for transmission lines:

		A	B	C	D	
Short line		1	Z	0	A	Length <80Km(66Kv) , Capacitance can be ignored , Parameters can be taken as lumped
Medium line	π	$1+\frac{ZY}{2}$	Z	$Y\left(1+\frac{ZY}{4}\right)$	A	Length 80-250Km , Capacitance - is line to neutral per Km , Parameters can be taken as lumped , Z- Total series impedance of line , Y – Total shunt admittance of line .
	T	$1+\frac{ZY}{2}$	$Z\left(1+\frac{ZY}{4}\right)$	Y	A	
Long line		$\cosh \gamma l$	$Z_c \sinh \gamma l$	$\frac{1}{Z_c}\sinh \gamma l$	A	Length >250Km , $\gamma = \sqrt{zy}$, $\gamma l = \sqrt{zlyl} = \sqrt{ZY}$, $Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{Z}{Y}}$, $Z = zl$; $Y = yl$, z–series impedance per unit length .

- Figure 1.7 shows the derivation of ABCD parameters for **nominal π method** in long transmission line.

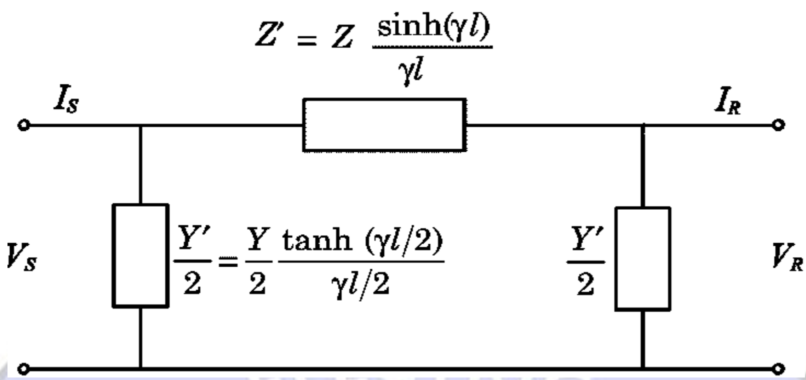


Fig. 1.7:

- The equation of sending-end voltage for **nominal π method** in long transmission line is;

$$V_S = \left(1 + \frac{Y'Z'}{2}\right) V_r + Z' I_r$$

- By comparing the above equation with the following one;

$$V_S = (\cosh \gamma l)\, V_r + Z_c (\sinh \gamma l)\, I_r$$

- We can get two terms. The first one is;

$$Z' = Z_C \sinh \gamma l$$

$$= \sqrt{z/y}\, \sinh \gamma l = zl \frac{\sinh \gamma l}{\sqrt{zy}\, l}$$

$$\therefore \quad Z' = Z\, \frac{\sinh \gamma l}{\gamma l}$$

Where, $Z = z\,l$ - Total series impedance of the line

- While the second term is;

$$1+\frac{Y'Z'}{2}=\cosh\gamma l$$

$$Y'/2=\frac{\cosh\gamma l-1}{Z'}$$

$$=\frac{\cosh\gamma l-1}{Z_C\sinh\gamma l}$$

$$=\frac{1}{Z_C}\tanh\frac{\gamma l}{2}$$

Where, $\tanh\frac{\gamma l}{2}=\frac{\cosh\gamma l-1}{\sinh\gamma l}$

$$\therefore\quad \frac{Y'}{2}=\frac{Y}{2}\left(\frac{\tanh\gamma l/2}{\gamma l/2}\right)$$

Where $Y = y\,l$ - Total shunt admittance of the line

Example 1.3:

A single-circuit 60-Hz transmission line is 370 Km (230 mi). The series impedance per unit length per phase is $0.8431\angle 79.04^\circ$ Ω/km and the shunt admittance per unit length per phase to neutral is $5.105\times10^{-6}\angle 90^\circ$ ℧/km. The load on the line is 125 MW at 215 KV with 100 % power factor. Find voltage, current, power at the sending end, and the voltage regulation.

Solution:

$$\gamma l = \sqrt{yz}\, l = 230\sqrt{0.8431 \times 5.105 \times 10^{-6}} \angle \frac{79.04^\circ + 90^\circ}{2}$$

$$= 0.4772 \angle 84.52^\circ = 0.0456 + j0.4750$$

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.8431}{5.105 \times 10^{-6}}} \angle \frac{79.04^\circ - 90^\circ}{2} = 406.4 \angle -5.48^\circ \ \Omega$$

$$V_R = \frac{215{,}000}{\sqrt{3}} = 124{,}130 \angle 0^\circ \ \text{V to neutral}$$

$$I_R = \frac{125{,}000{,}000}{\sqrt{3} \times 215{,}000} = 335.7 \angle 0^\circ \ \text{A}$$

noting that 0.4750 rad = 27.22°

$$\cosh \gamma l = \frac{1}{2}\varepsilon^{0.0456} \angle 27.22^\circ + \frac{1}{2}\varepsilon^{-0.0456} \angle -27.22^\circ$$

$$= 0.4654 + j0.2394 + 0.4248 - j0.2185$$

$$= 0.8902 + j0.0209 = 0.8904 \angle 1.34^\circ$$

$$\sinh \gamma l = 0.4654 + j0.2394 - 0.4248 + j0.2185$$

$$= 0.0406 + j0.4579 = 0.4597 \angle 84.93^\circ$$

- By using the following equation;

$$V_S = (\cosh \gamma l)\, V_r + Z_c (\sinh \gamma l)\, I_r$$

$$I_S = \frac{1}{Z_c}(\sinh \gamma l)\, V_r + (\cosh \gamma l)\, I_r$$

$$V_S = 124{,}130 \times 0.8904\angle 1.34^\circ + 335.7 \times 406.4\angle -5.48^\circ \times 0.4597\angle 84.93^\circ$$

$$= 110{,}495 + j2{,}585 + 11{,}483 + j61{,}656$$

$$= 137{,}860\angle 27.77^\circ \text{ V}$$

$$I_S = 335.7 \times 0.8904\angle 1.34^\circ + \frac{124{,}130}{406.4\angle -5.48^\circ} \times 0.4597\angle 84.93^\circ$$

$$= 298.83 + j6.99 - 1.00 + j140.41$$

$$= 332.31\angle 26.33^\circ \text{ A}$$

At the sending end

$$\text{Line voltage} = \sqrt{3} \times 137.86 = 238.8 \text{ kV}$$

$$\text{Line current} = 332.3 \text{ A}$$

$$\text{Power factor} = \cos(27.77^\circ - 26.33^\circ) = 0.9997 \cong 1.0$$

$$\text{Power} = \sqrt{3} \times 238.8 \times 332.3 \times 1.0 = 137{,}443 \text{ kW}$$

we see that at no load ($I_R = 0$)

$$V_R = \frac{V_S}{\cosh \gamma l}$$

So, the voltage regulation is

$$\frac{137.86/0.8904 - 124.13}{124.13} \times 100 = 24.7\%$$

Homework:

Find the same requirements for the above example by using π method.

1.3- Two Transmission Lines in Cascaded (Series):

- Figure 1.8 shows two transmission lines in series. The purpose of this connection is to find the relationships between the sending-end and receiving-end in terms of voltage and currents using ABCD method.

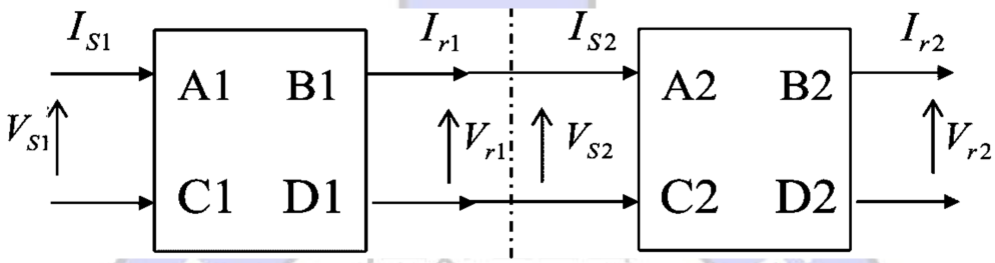


Fig. 1.8:

- For the first block, the equations in terms of voltage and current are;

$$V_{S1} = A1\,V_{r1} + B1\,I_{r1}$$
$$I_{S1} = C1\,V_{r1} + D1\,I_{r1}$$

- The matrix form of the above equations is:

$$\begin{bmatrix} V_{S1} \\ I_{S1} \end{bmatrix} = \begin{bmatrix} \mathrm{A1} & \mathrm{B1} \\ \mathrm{C1} & \mathrm{D1} \end{bmatrix} \begin{bmatrix} V_{r1} \\ I_{r1} \end{bmatrix}$$

- For the second block, the equations in terms of voltage and current are;

$$V_{S2} = A2\,V_{r2} + B2\,I_{r2}$$
$$I_{S2} = C2\,V_{r2} + D2\,I_{r2}$$

- The matrix form of the above equations is:

$$\begin{bmatrix} V_{S2} \\ I_{S2} \end{bmatrix} = \begin{bmatrix} \mathrm{A2} & \mathrm{B2} \\ \mathrm{C2} & \mathrm{D2} \end{bmatrix} \begin{bmatrix} V_{r2} \\ I_{r2} \end{bmatrix}$$

Where;

$$V_{r1} = V_{S2} \quad \text{and} \quad I_{r1} = I_{S2}$$

- The combination of the two matrix forms is;

$$\begin{bmatrix} V_{S1} \\ I_{S1} \end{bmatrix} = \begin{bmatrix} \mathrm{A1} & \mathrm{B1} \\ \mathrm{C1} & \mathrm{D1} \end{bmatrix} \begin{bmatrix} \mathrm{A2} & \mathrm{B2} \\ \mathrm{C2} & \mathrm{D2} \end{bmatrix} \begin{bmatrix} V_{r2} \\ I_{r2} \end{bmatrix}$$

- The result of the above matrix equations is;

$$\begin{bmatrix} V_{S1} \\ I_{S1} \end{bmatrix} = \begin{bmatrix} \mathrm{A1A2+B1C2} & \mathrm{A1B2+B1D2} \\ \mathrm{C1A2+C2D1} & \mathrm{B2C1+D1D2} \end{bmatrix} \begin{bmatrix} V_{r2} \\ I_{r2} \end{bmatrix}$$

- The final equation between the sending-end and the receiving-end is;

$$\begin{bmatrix} V_{S1} \\ I_{S1} \end{bmatrix} = \begin{bmatrix} \mathrm{A_o} & \mathrm{B_o} \\ \mathrm{C_o} & \mathrm{D_o} \end{bmatrix} \begin{bmatrix} V_{r2} \\ I_{r2} \end{bmatrix}$$

1.4- Power Flow through a Transmission Line

- A major section of power system engineering deals in the transmission of electrical power from one particular place (e.g. generating station) to another one (e.g. distribution units) with maximum efficiency.
- Although power flow at any point along a transmission line can always be found if the voltage, current, and power factor are known or can be calculated, very interesting equations for power can be derived in terms of ABCD constants between sending-and receiving-ends.

1.4.1- Receiving-end Circle Diagram:

- The power flow in a transmission line can be calculated by considering the system shown in figure 1.9. It consists of a single transmission line between two buses. These buses are sending end bus and receiving end bus.

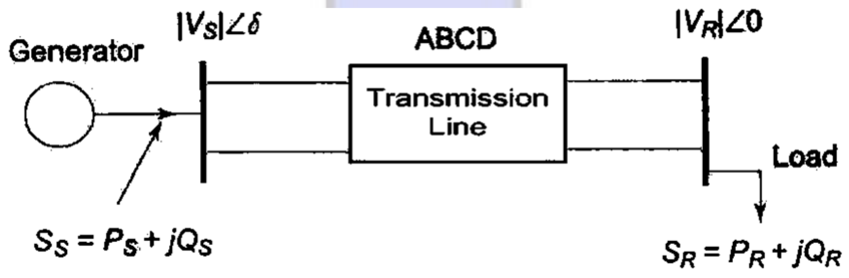


Fig. 1.9:

- Refer to figure 1.9, the complex power leaving the receiving-end of the transmission line can be expressed as (on per phase basis);

$$S_R = P_R + jQ_R = V_R I_R^* \qquad (3.37)$$

- As we know that the voltage sending-end equation for the transmission line in terms of ABCD constants are;

$$V_S = AV_R + BI_R \qquad (3.38)$$

- From equation 3.38, receiving-end current can be expressed in terms of receiving-end and sending-end voltages as;

$$I_R = \frac{V_S - AV_R}{B} \qquad (3.40)$$

$$I_R = \frac{1}{B} V_S - \frac{A}{B} V_R$$

- Letting;

$$A = |A|\angle\alpha \qquad B = |B|\angle\beta$$

$$V_R = |V_R|\angle 0° \qquad V_S = |V_S|\angle\delta$$

- Substituting above quantities in equation 3.40, we can get;

$$\vec{I}_r = \left(\frac{V_S}{B}\angle\delta - \beta\right) - \left(\frac{AV_r}{B}\angle\alpha - \beta\right) \qquad (3.41)$$

- From equation 3.41, the conjugate of I_r is;

$$\vec{I}_r^{\,*} = \left(\frac{V_S}{B}\angle\beta-\delta\right)-\left(\frac{A\,V_r}{B}\angle\beta-\alpha\right) \qquad (3.42)$$

- Refer to equation 3.37, the complex power at the receiving end (the volt-amperes delivered to the load) is given by;

$$S_R = \frac{|V_R||V_S|}{|B|}\angle(\beta-\delta)-\frac{|V_R|^2|A|}{|B|}\angle(\beta-\alpha) \qquad (3.43)$$

Where;

$$S_R = P_R + jQ_R$$

- The real and reactive power (from complex power equation 3.43) at the receiving end are;

$$P_R = \frac{|V_R||V_S|}{|B|}Cos(\beta-\delta)-\frac{|V_R|^2|A|}{|B|}Cos(\beta-\alpha) \qquad (3.44)$$

$$Q_R = \frac{|V_R||V_S|}{|B|}Sin(\beta-\delta)-\frac{|V_R|^2|A|}{|B|}Sin(\beta-\alpha) \qquad (3.45)$$

- It is easy to see from equation 3.44 that the received power P_R will be maximum at;

$$\beta-\delta=0$$

$$or \quad \beta=\delta$$

$$\therefore \quad P_{r\,(\max.)} = \frac{|V_R||V_S|}{|B|} - \frac{|V_R|^2|A|}{|B|}Cos(\beta-\alpha)$$

- While the corresponding Q_R at max P_R is;

$$Q_R = -\frac{|V_R|^2|A|}{|B|}Sin(\beta-\alpha)$$

- Now, we will see how to represent characteristics of transmission line graphically as a circle diagram by taking V_s, V_r, I_s, and I_r as a reference.
- A circle diagram is drawn with real power P on X-axis and Q on Y-axis on complex plane.
- To draw the circle diagram, the complex power at the receiving-end is;

$$S_R = P_R + jQ_R$$

$$S_R = \frac{|V_R||V_S|}{|B|}\angle(\beta-\delta) - \frac{|V_R|^2|A|}{|B|}\angle(\beta-\alpha)$$

$$\frac{|V_R|^2|A|}{|B|}\angle(\beta-\alpha) = P_R + jQ_R + \frac{|V_R||V_S|}{|B|}\angle(\beta-\delta)$$

- Thus, the circle diagram of the receiving-end power is;

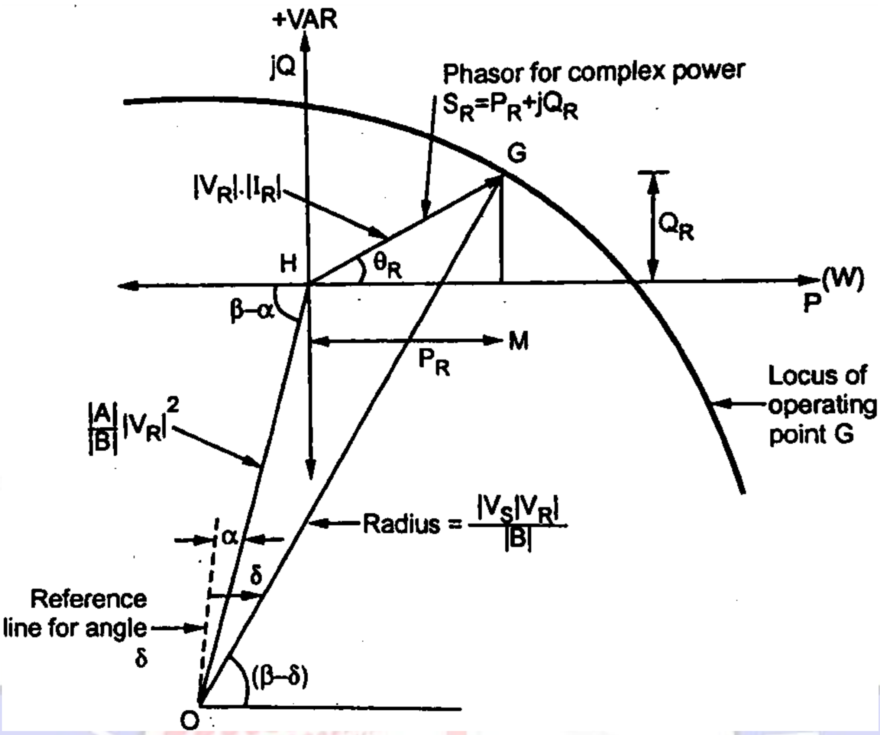


Fig. 1.10:

- This diagram is useful to determine the active power (P_r), reactive power (Q_r), and power angle (δ).
- The θ_r is the phase angle by which V_r leads I_r. The position of point O is independent of load current (I_r) and will change as long as V_r is constant.
- The distance OG remains constant if the values of V_s and V_r are constant.
- The distance HG goes on changing when the load is changed.
- Any change in P_r will require a change in $Q_r$to keep point G on the circle.

- If different values of V_s are held constant for the same value of V_r, the location of point O is unchanged but a new circle with different radius (OG) is obtained as shown in figure 1.11.

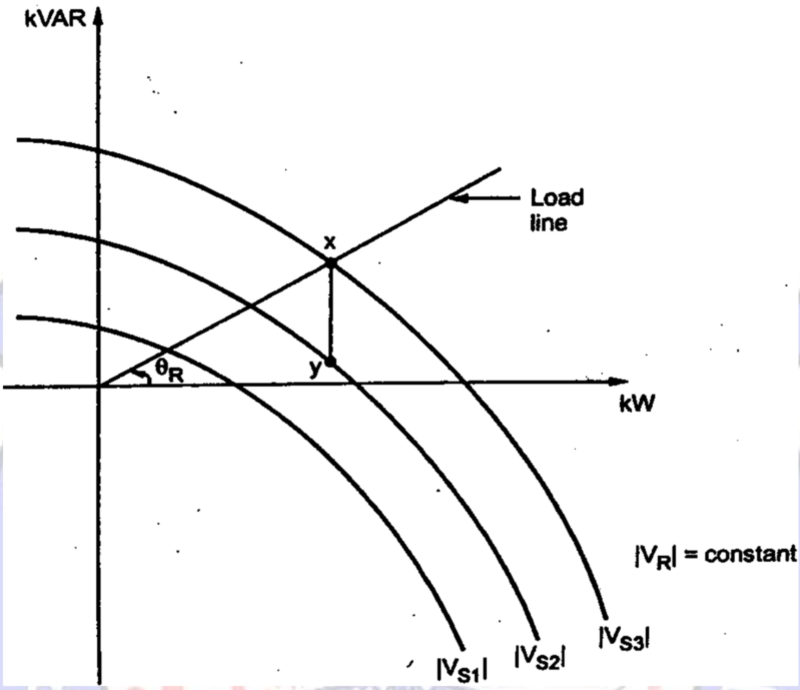


Fig. 1.11:

- If a vertical line is drawn from x on circle with sending end voltage V_{S3} to the point y on circle with sending end voltage V_{S2} as shown in figure 1.11, then the distance xy represents the amount of negative reactive power that must be drawn by capacitors added in parallel with the load to maintain constant V_r when sending end voltage is reduced from V_{S3} to V_{S2}.

1.4.2- Sending-end Circle Diagram

- Refer to figure 1.9, the complex power entering the sending-end of the transmission line can be expressed as (on per phase basis);

$$S_S = P_S + jQ_S = V_S I_S^* \quad (3.46)$$

- As we know that the sending-end current equation for the transmission line in terms of ABCD constants are;

$$I_S = CV_R + DI_R \quad (3.47)$$

- Letting;

$$C = |C| \angle \gamma$$

$$A = D = |D| \angle \alpha$$

- Refer to equation 3.41, the receiving-end current is;

$$\vec{I}_r = \left(\frac{V_S}{B}\angle\delta-\beta\right) - \left(\frac{AV_r}{B}\angle\alpha-\beta\right)$$

- By substituting the above quantities in equation 3.47, the sending end current will be;

$$I_s = C\angle\gamma \cdot V_r\angle 0 + D\angle\alpha\left[\frac{V_s}{B}\angle\delta-\beta - \frac{AV_r}{B}\angle\alpha-\beta\right]$$

$$I_s = CV_r\angle\gamma + \frac{DV_s}{B}\angle\delta-\beta+\alpha - \frac{ADV_r}{B}\angle\alpha-\beta+\alpha$$

$$I_s = V_r\left[C\angle\gamma - \frac{AD}{B\angle\beta}\angle 2\alpha\right] + \frac{DV_s}{B}\angle\delta-\beta+\alpha$$

$$I_s = V_r\left[\frac{C\angle\gamma\, B\angle\beta - AD\angle 2\alpha}{B\angle\beta}\right] + \frac{DV_s}{B}\angle\delta-\beta+\alpha$$

Where; $AD - BC = 1 \Rightarrow BC - AD = -1$

$$\therefore I_s = \frac{DV_s}{B}\angle\delta-\beta+\alpha - \frac{V_r}{B}\angle -\beta \qquad (3.48)$$

- By substituting the equation 3.48 in equation 3.46, the complex power at the sending end will be;

$$S_s = V_s I_s^{*} = V_s\angle\delta\left[\frac{DV_s}{B}\angle\beta-\delta-\alpha - \frac{V_r}{B}\angle\beta\right]$$

$$= \frac{DV_s^2}{B}\angle\delta+\beta-\delta-\alpha - \frac{V_r}{B}\angle\delta+\beta$$

$$S_s = \frac{DV_s^2}{B}\angle\beta-\alpha - \frac{V_s V_r}{B}\angle\beta+\delta \qquad (3.49)$$

- The real and reactive power (from complex power equation 3.46) at the sending end are;

$$P_S = \left|\frac{D}{B}\right| |V_S|^2 \cos(\beta-\alpha) - \frac{|V_S||V_R|}{|B|}\cos(\beta+\delta) \qquad (5.50)$$

$$Q_S = \left|\frac{D}{B}\right| |V_S|^2 \sin(\beta - \alpha) - \frac{|V_S||V_R|}{|B|}\sin(\beta + \delta) \qquad (5.51)$$

- To draw the circle diagram, the complex power at the receiving-end is;

$$S_S = P_S + jQ_S$$

$$S_s = \frac{DV_s^2}{B}\angle\beta-\alpha - \frac{V_s V_r}{B}\angle\beta+\delta$$

$$\frac{DV_s^2}{B}\angle\beta-\alpha = P_S + jQ_S + \frac{V_s V_r}{B}\angle\beta+\delta$$

- Thus, the circle diagram of the sending-end power is;

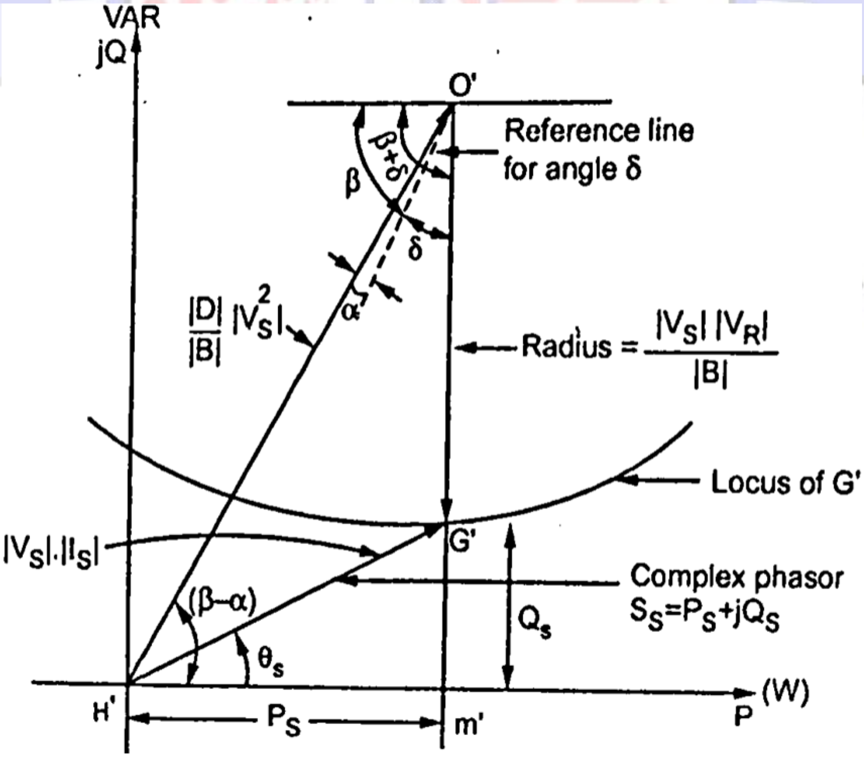


Fig. 1.12:

Example 1.4:

The constants of 3-phase line are $A = 0.9\angle 2°$ and $B = 140\angle 70°$ Ω/ph. The line delivers 60 mvA at 132 KV and 0.8 power factor lag. Draw circle diagrams and find (a) sending end voltage and power angle (b) maximum power which the line can delivered with above values of sending and receiving end voltages (c) sending end power and power factor (d) line losses.

Sol:

(a)

- Refer to figure 1.10, the center $(HO) = \frac{|A|\,|V_r|^2}{|B|} = \frac{0.9*132^2}{140} = 112$ mvA and $\angle\beta - \alpha = 70° - 2° = 68°$
- Selecting a scale of 40 mvA for each 1 cm, so the $HO = 112/40 = 2.8\ cm$ and 68° w.r.t -ve x-axis.
- Draw HG at $\theta_r = cos^{-1}(0.8) = 36.87°$, where $HG = S_r = 60$ mvA then the length of HG is 60/40=1.5 cm
- Draw the receiving end circle with HO as a center and OG as a radius, by measure OG, it will be found as 4.2 cm or 4.2*40=168 mvA
- Then, the radius $OG = \frac{|V_s|\,|V_r|}{|B|}$, by multiply both sides we can get

$$|V_s| = \frac{168 * 140}{132} = 178.2\ KV$$

- Draw the reference line at an angle of $\propto$ before HO line and calculate the power angle (δ) which is the angle between the reference line and OG line, by measuring; $\delta = 13°$.
- The above procedures are shown by the graph below.

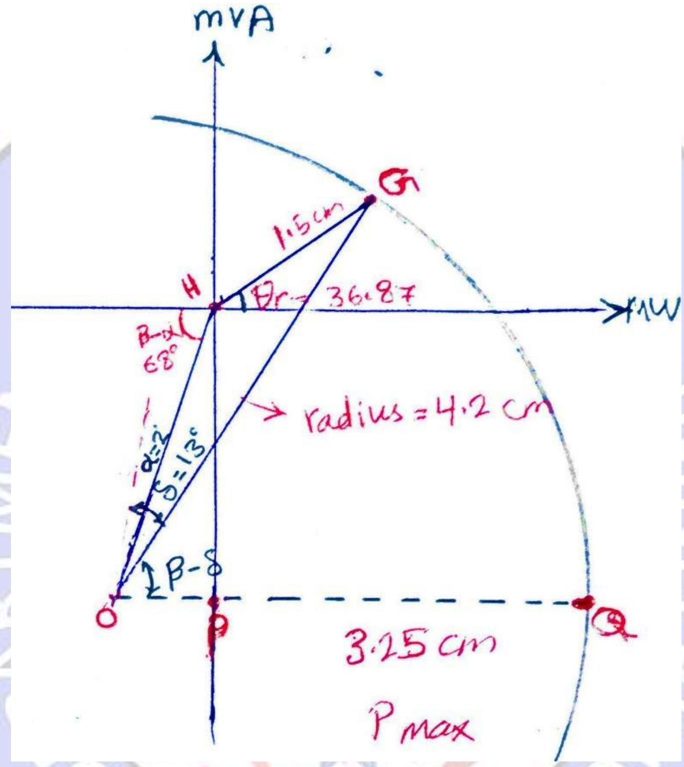


Fig. 1.13:

For your information;

$$\vec{S}_r = |S| \angle\theta_r$$
$$= V_r\, I_r \angle\theta_r$$
$$= P_r + j\, Q_r$$
$$= V_r\, I_r \cos\theta_r + j\, V_r\, I_r \sin\theta_r$$

(b)

- Maximum power, with $V_r = 132\ KV$and $V_s = 178.2\ KV$, is PQ line as shown in figure 1.13 which is 3.25 cm by measuring or 3.25*40=130 MW.

(C)

- Refer to figure 1.12, the center $(H'O')$ at ∠68° w.r.t $+ve\ x-axis$, the $H'O' = \frac{|A|\ |V_S|^2}{|B|} = \frac{0.9*178.2^2}{140} = 204$ mvA or 204/40=5.1 cm.
- Radius of sending end circle is $O'G' = \frac{|V_s|\ |V_r|}{|B|} = 168$ mvA or 168/40=4.2 cm.
- Draw sending end circle with $H'O' = 5.1\ cm$ as a center.
- Draw the reference line at ∝=2° after the line $H'O'$.
- Measure the power angle $\delta = 13°$, which is the angle between the reference line and the $O'G'$ line, then draw the $O'G' = 4.2$ cm as a radius.
- Draw the $H'G'$ line, which represents the sending end power (S_s). By measuring, the $H'G'$ is 1.5 cm or 1.5*40=60 mvA.
- Measure the $\theta_s = 20°$, which is the angle between $H'G'$ line and the $+ve\ x-axis$.
- Sending end power factor $= \cos(\theta_s) = \ cos(20) = 0.94$ lag.

- Measure the $H'M'$=1.46 cm or 1.46*40=58.4 MW, which represents the active sending power (P_s).

(d)

- Line losses = $P_s - P_r = 58.4 - S_r * p.f = 58.4 - 60 * 0.8 = 10.4$ MW
- The above procedures are shown by the graph below.

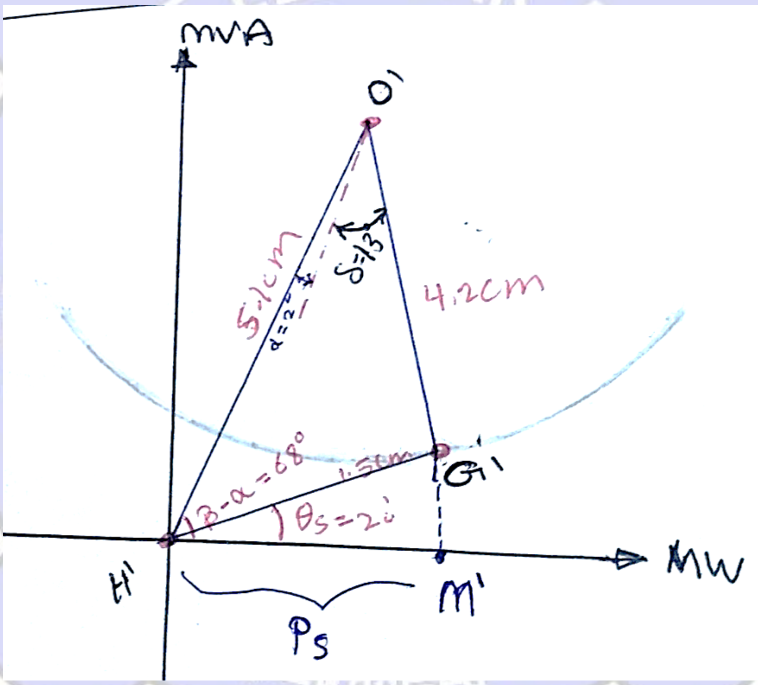


Fig. 1.14:

Chapter Two

Overhead Transmission Lines (Mechanical Design)

2.1- Introduction:

- Electric power can be transmitted by means of underground cables or by overhead lines as can be seen below figure 2.1.

Fig. 2.1: From google

- The underground cables are rarely used for power transmission due to two main reasons.
- Firstly, power is generally transmitted over long distances to load centers. Obviously, the installation costs for underground transmission line will be very heavy.
- Secondly, electric power has to be transmitted at high voltages for economic reasons. It is very difficult to provide proper insulation to the cables to withstand such higher pressures.
- Therefore, power trasmission over long distances is carried out by using overhead lines.

- An overhead line is subjected to uncertain weather conditions, this calls for the use of proper mechanical factors of safety in order to ensure the continuity of operation in the line.

2.2- Main Components of Overhead Lines:

- In general, figure 2.2 shows the main components of an overhead transmission line as listed below:

  a- **Conductors** which carry electric power from the sending end station to the receiving end station.

  b- **Supports** which may be poles or towers and keep the conductors at a suitable level above the ground.

  c- **Insulators** which are attached to supports and insulate the conductors from the ground.

  d- **Cross arms** which provide support to the insulators.

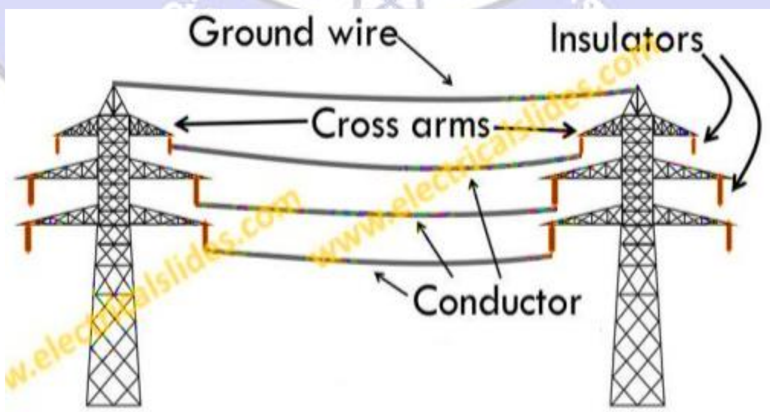


Fig. 2.2: From google

2.3- Conductor Materials:

- The conductor material used for transmission and distribution of electric power should have the following properties:

  (i) high electrical conductivity.

  (ii) high tensile strength in order to withstand mechanical stresses.

  (iii) low cost so that it can be used for long distances.

- The choice of a material will depend upon the cost, the required electrical and mechanical properties and the local conditions.
- The most commonly used conductor materials for overhead lines are copper and aluminium.
- Copper is an ideal material for overhead lines which has a high electrical conductivity and greater tensile strength.
- Aluminium is cheap and light as compared to copper but it has much smaller conductivity and tensile strength.

2.4- Line Supports:

- The supporting structures for overhead line conductors are various types of poles and towers called line supports. In general, the line supports should have the following properties:

  (i) High mechanical strength to withstand the weight of conductors and wind loads etc.

(ii) Light in weight without the loss of mechanical strength.

(iii) Cheap in cost.

(iv) Longer life.

- The purpose of using line supports is to (1) provide the required clearance between conductors and ground (2) carry conductors and insulators.
- The line supports used for transmission and distribution of electric power are of various types including wooden poles, R.C.C. poles, and steel towers.

- **Wooden Poles:**
    - These are made of seasoned wood such **(sal)** type.
    - They are suitable for lines which has shorter spans, say upto 50 meter, and upto 11 KV.
    - These supports are cheap, therefore widely used for distribution purposes.
    - The wooden poles generally tend to rot below the ground level, causing foundation failure. In order to prevent this, the portion of the pole below the ground level is impregnated with preservative compounds like creosote oil.
    - Double pole structures of the 'A' or 'H' type are often used (See figure 2.3).

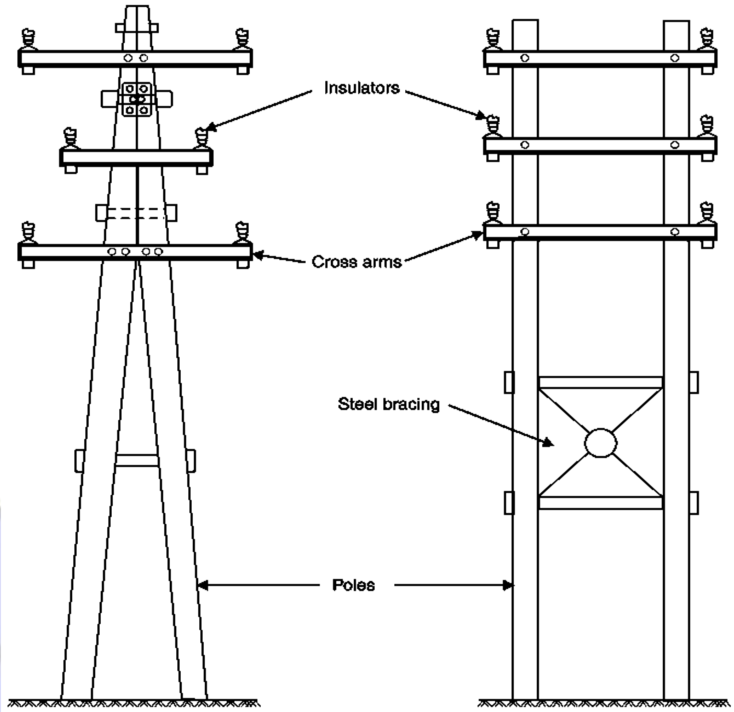


Fig. 2.3:

From google

- The main objections to wooden supports are: (i) tendency to rot below the ground level (ii) cannot be used for voltages higher than 20 kV (iii) less mechanical strength and (iv) require periodical inspection.

- **R.C.C. Poles:**
  - ❖ The reinforced concrete poles have greater mechanical strength, longer life, upto to 33 KV, and longer spans than wooden poles.
  - ❖ Require little maintenance.
  - ❖ Figure 2.4 shows R.C.C. poles for single and double circuit.

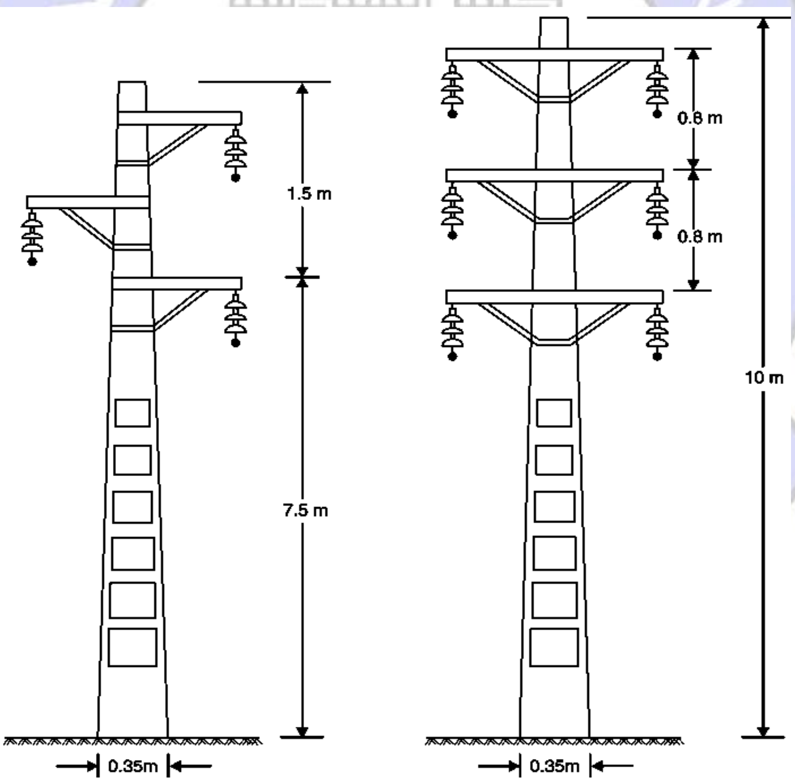


Fig. 2.4:

❖ The main difficulty with the use of these poles is the high cost of transport due to their heavy weight. Therefore, they are often manufactured at the site in order to avoid heavy cost of transportation.

From google

- **Steel Towers:**

❖ In practice, wooden and reinforced concrete poles are used for distribution purposes at low voltages, say upto 11 and 33 kV.

❖ However, for long distance transmission at higher voltage, steel towers are employed (more than 66 KV).

❖ Steel towers have greater mechanical strength, longer life, and longer spans.

❖ Figure 2.5 shows a single circuit and double circuit towers.

❖ The double circuit has the advantage that it ensures continuity of supply. If there is a breakdown of one circuit, the continuity of supply can be maintained by the other circuit.

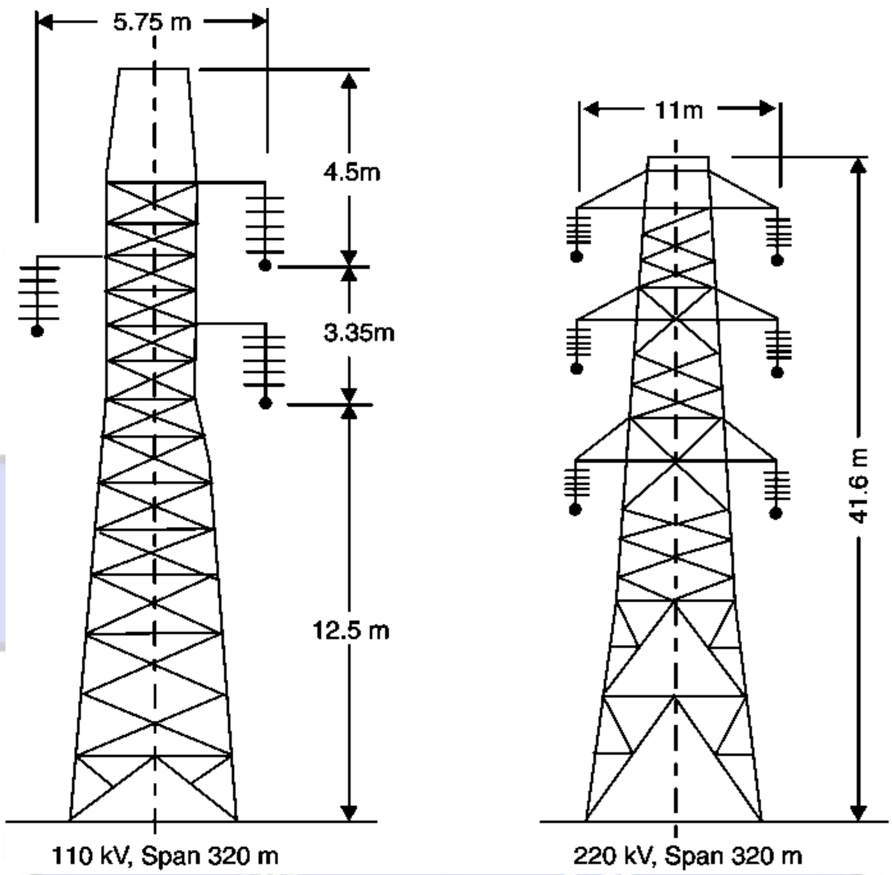
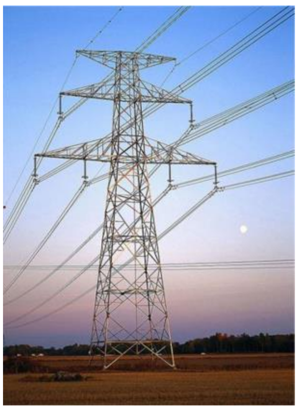


Fig. 2.5:



2.5- Insulators:

- The overhead line conductors should be supported on the poles or towers in such a way that currents from conductors do not flow to earth through supports. This is achieved by insulators.
- The purpose of the insulators is (1) to provide necessary insulation between line conductors and supports and (2) to prevent any leakage current from conductors to earth.
- In general, the insulators should have the following desirable properties:
  - ❖ (i) High mechanical strength in order to withstand conductor load, wind load etc.
  - ❖ (ii) High electrical resistance of insulator material in order to avoid leakage currents to earth.
  - ❖ (iii) High ratio of puncture strength to flashover.

2.5.1- Types of Insulators:

- The successful operation of an overhead line depends upon the proper selection of insulators.

(i) Pin Type Insulator:

- ❖ Figure 2.6 and 2.7 show the theoretical and practical picture of the pin type insulator. The pin type insulator is secured to the cross-arm on the pole.

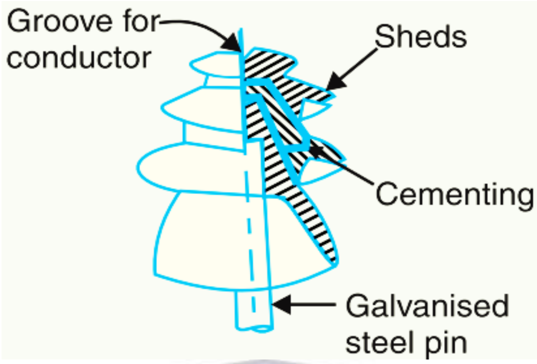


Fig. 2.6:

- There is a groove on the upper end of the insulator for housing the conductor. The conductor passes through this groove and is bound by the annealed wire of the same material as the conductor.

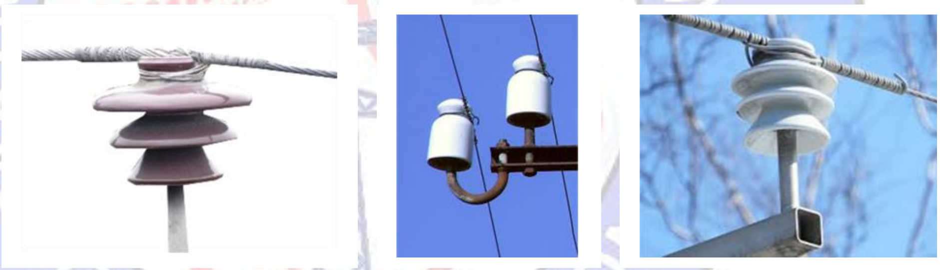


Fig. 2.7: From google

- Pin type insulators are used for transmission and distribution of electric power at voltages upto 33 kV.
- Causes of insulator failure are flash-over and puncture due to the mechanical and electrical stresses as shown in figure 2.8.

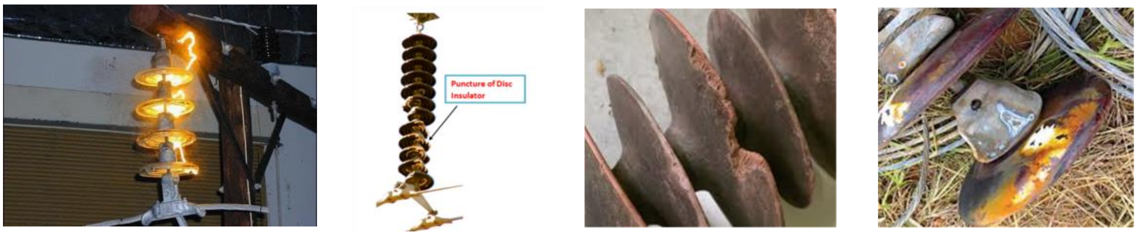


Fig. 2.8: From google

- Safety factor of insulator is the ratio of puncture strength to flashover voltage;

$$\text{Safety factor of insulator} = \frac{\text{Puncture strength}}{\text{Flash - over voltage}}$$

- For pin type insulators, the value of safety factor is about 10.

(ii) Strain Insulators:

- When there is a dead end of the line or there is corner or sharp curve, the line is subjected to greater tension. In order to relieve the line of excessive tension, strain insulators are used as shown in figure 2.9.
- The discs of strain insulators are used in the vertical plane.

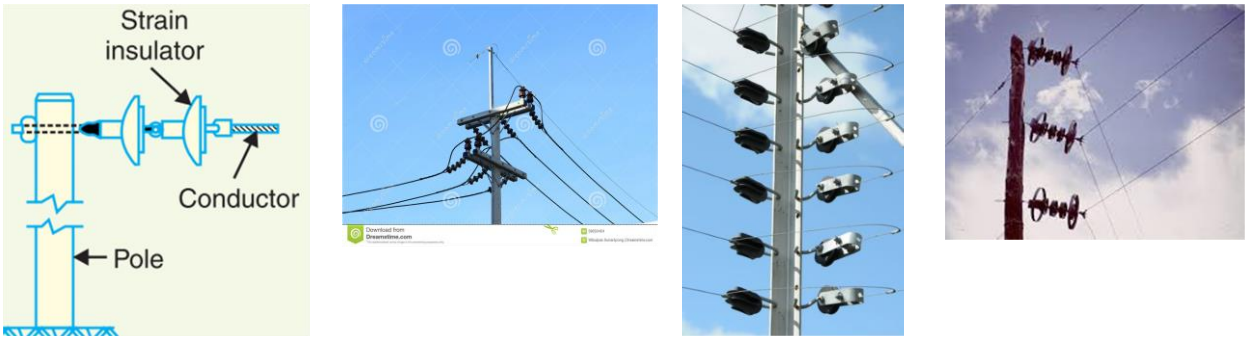


Fig. 2.9:

(iii) Suspension Type Insulator:

❖ The cost of pin type insulator increases rapidly as the working voltage is increased. Therefore, this type of insulator is not economical beyond 33 kV. For high voltages (>33 kV), it is a usual practice to use suspension type insulators shown in Fig. 2.10.

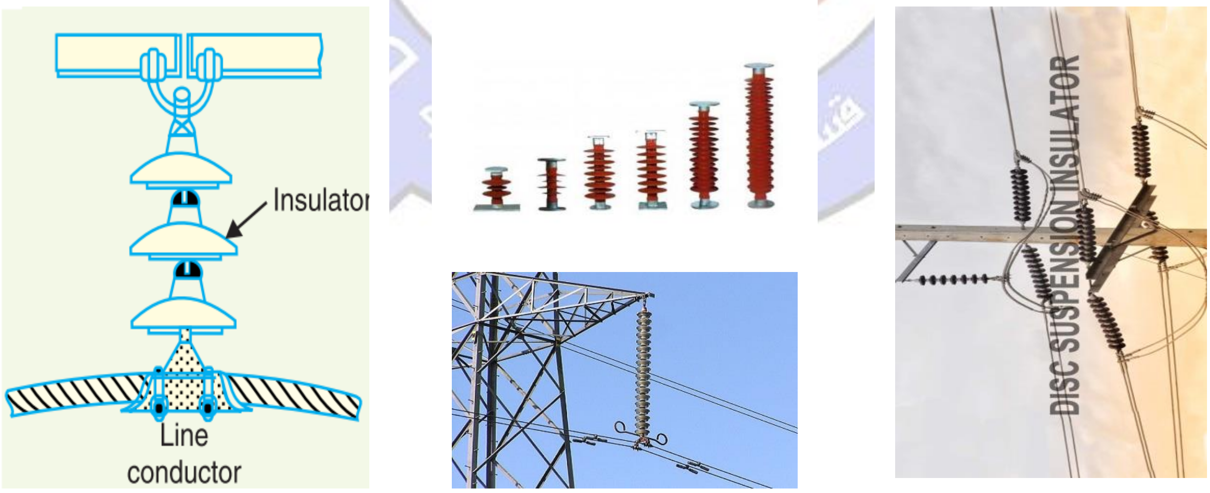


Fig. 2.10:

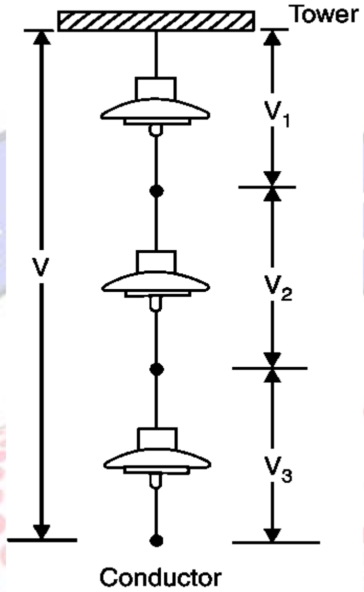
- In suspension insulator numbers of insulators are connected in series to form a string, each unit or disc is designed for low voltage, say 11 kV.
- The number of discs in series would obviously depend upon the working voltage. For instance, if the working voltage is 66 kV, then six discs in series will be provided on the string.

Advantages of Suspension insulators:

- Suspension type insulators are cheaper than pin type insulators for voltages beyond 33 kV.
- Each disc of suspension type insulator is designed for low voltage, usually 11 kV.
- If any disc is damaged, no need to replace the whole string but the damaged one can be replaced.
- The suspension type insulators are generally used with steel towers.

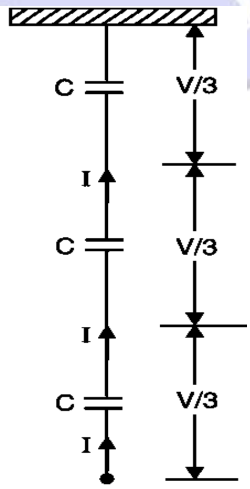
2.5.2- Potential Distribution over Suspension Insulator String:

- A string of suspension insulators consists of a number of porcelain discs connected in series.
- Fig. 2.11(i) shows 3-disc string of suspension insulators.



(*i*)

- Therefore, each disc forms a capacitor *C* as shown in Fig. 2.11 (ii). This is known as mutual capacitance or self-capacitance.



(*ii*)

- If there were mutual capacitance alone, then charging current would have been the same through all the discs and consequently voltage across each unit would have been the same i.e., V/3.
- However, in actual practice, capacitance also exists between metal fitting of each disc and tower or earth. This is known as shunt capacitance C_1 as shown in Fig. 2.11 (iii).

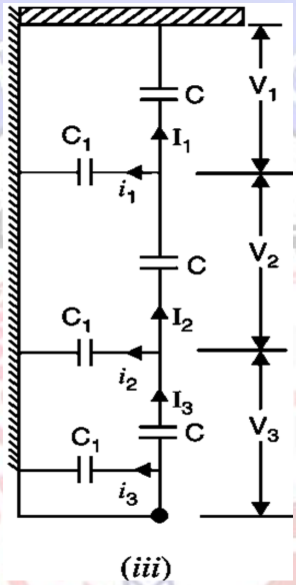


Fig. 2.11

- Due to shunt capacitance, charging current is not the same through all the discs of the string. Therefore, voltage across each disc will be different.
- Obviously, the disc nearest to the line conductor will have the maximum voltage.
- Thus, V_3 will be much more than V_2 or V_1

The following points may be noted regarding the potential distribution over a string of suspension insulators:

(i) The voltage impressed on a string of suspension insulators does not distribute itself uniformly across the individual discs due to the presence of shunt capacitance.

(ii) The disc nearest to the conductor has maximum voltage across it. As we move towards the cross-arm, the voltage across each disc goes on decreasing.

(iii) The unit nearest to the conductor is under maximum electrical stress and is likely to be punctured. Therefore, means must be provided to equalize the potential across each unit.

The mathematical expression regarding the potential distribution over a string of suspension insulators:

- Fig. 2.12 shows the equivalent circuit for a 3-disc string.
- Let us suppose that self-capacitance of each disc is C.
- Let us further assume that shunt capacitance C_1 is some fraction K of self-capacitance i.e., $C_1 = KC$.
- Starting from the cross-arm or tower, the voltage across each unit is V_1, V_2 and V_3 respectively.

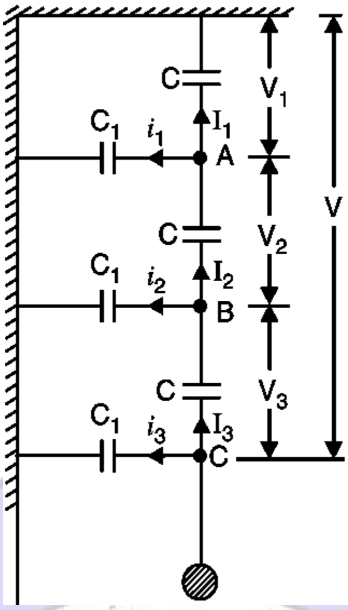


Fig. 2.12:

- Applying Kirchhoff's current law to node A, we get;

$$I_2 = I_1 + i_1$$

or $$V_2 \omega C^* = V_1 \omega C + V_1 \omega C_1$$

or $$V_2 \omega C = V_1 \omega C + V_1 \omega K C$$

$\therefore$ $$V_2 = V_1 (1 + K) \qquad ...(i)$$

- Applying Kirchhoff's current law to node B, we get;

$$I_3 = I_2 + i_2$$

or $$V_3 \omega C = V_2 \omega C + (V_1 + V_2)\, \omega C_1 \dagger$$

or $$V_3 \omega C = V_2 \omega C + (V_1 + V_2)\, \omega K C$$

or $$V_3 = V_2 + (V_1 + V_2)K$$

$$= KV_1 + V_2 (1 + K)$$

$$= KV_1 + V_1 (1 + K)^2$$

$$= V_1 [K + (1 + K)^2]$$

$\therefore$ $$V_3 = V_1 [1 + 3K + K^2] \qquad ...(ii)$$

- Voltage between conductor and earth (i.e., tower) is

$$\begin{aligned} V &= V_1 + V_2 + V_3 \\ &= V_1 + V_1(1+K) + V_1(1+3K+K^2) \\ &= V_1(3+4K+K^2) \end{aligned}$$

$$\therefore \quad V = V_1(1+K)(3+K) \qquad \ldots(iii)$$

From expressions (*i*), (*ii*) and (*iii*), we get,

$$\frac{V_1}{1} = \frac{V_2}{1+K} = \frac{V_3}{1+3K+K^2} = \frac{V}{(1+K)(3+K)} \qquad \ldots(iv)$$

$$\therefore \text{Voltage across top unit,} \quad V_1 = \frac{V}{(1+K)(3+K)}$$

$$\text{Voltage across second unit from top,} \quad V_2 = V_1(1+K)$$

$$\text{Voltage across third unit from top,} \quad V_3 = V_1(1+3K+K^2)$$

The following points may be noted from the above mathematical analysis:

(i) If $K = 0 \cdot 2$ (Say), then from exp. (iv), we get, $V_2 = 1 \cdot 2\, V_1$ and V_3= 1·64 V_1. This clearly shows that disc nearest to the conductor has maximum voltage across it; the voltage across other discs decreasing progressively as the cross-arm in approached.

(ii) The greater the value of $(K = C_1/C)$, the more non-uniform is the potential across the discs and lesser is the string efficiency.

(iii) The inequality in voltage distribution increases with the increase of number of discs in the string. Therefore, shorter string has more efficiency than the larger one.

2.5.3- String Efficiency:

- As stated above, the voltage applied across the string of suspension insulators is not uniformly distributed across various units or discs.
- The disc nearest to the conductor has much higher potential than the other discs.
- This unequal potential distribution is undesirable and is usually expressed in terms of string efficiency.
- The ratio of voltage across the whole string to the product of number of discs and the voltage across the disc nearest to the conductor is known as string efficiency i.e.,

$$\text{String efficiency} = \frac{\text{Voltage across the string}}{n \times \text{Voltage across disc nearest to conductor}}$$

$$n = \text{number of discs in the string.}$$

- String efficiency is an important consideration since it decides the potential distribution along the string.
- The greater the string efficiency, the more uniform is the voltage distribution.
- Thus 100% string efficiency is an ideal case for which the voltage across each disc will be exactly the same.
- Although it is impossible to achieve 100% string efficiency, yet efforts should be made to improve it as close to this value as possible.

Example 2.1:

In a 33 kV overhead line, there are three units in the string of insulators. If the capacitance between each insulator pin and earth is 11% of self-capacitance of each insulator, find (i) the distribution of voltage over 3 insulators and (ii) string efficiency.

Solution. Fig. 8.14. shows the equivalent circuit of string insulators. Let V_1, V_2 and V_3 be the voltage across top, middle and bottom unit respectively. If C is the self-capacitance of each unit, then KC will be the shunt capacitance.

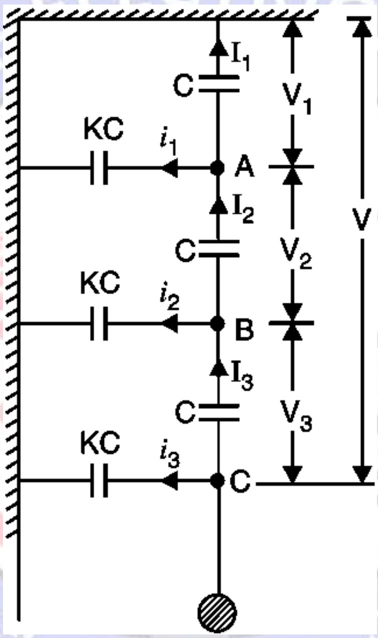


Fig. 8.14

$$K = \frac{\text{Shunt Capacitance}}{\text{Self-capacitance}} = 0.11$$

$$\text{Voltage across string, } V = 33/\sqrt{3} = 19{\cdot}05 \text{ kV}$$

At Junction A

$$I_2 = I_1 + i_1$$

or $$V_2 \omega C = V_1 \omega C + V_1 K \omega C$$

or $$V_2 = V_1 (1 + K) = V_1 (1 + 0{\cdot}11)$$

or $$V_2 = 1{\cdot}11 \, V_1 \quad ...(i)$$

At Junction B

$$I_3 = I_2 + i_2$$

or $$V_3 \omega C = V_2 \omega C + (V_1 + V_2) K \omega C$$

or $$V_3 = V_2 + (V_1 + V_2) K$$

$$= 1{\cdot}11 V_1 + (V_1 + 1{\cdot}11 \, V_1) \, 0{\cdot}11$$

$\therefore$ $$V_3 = 1{\cdot}342 \, V_1$$

(*i*) Voltage across the whole string is

$$V = V_1 + V_2 + V_3 = V_1 + 1{\cdot}11 \, V_1 + 1{\cdot}342 \, V_1 = 3{\cdot}452 \, V_1$$

or $$19{\cdot}05 = 3{\cdot}452 \, V_1$$

$\therefore$ Voltage across top unit, $V_1 = 19{\cdot}05/3{\cdot}452 = \mathbf{5{\cdot}52\ kV}$

Voltage across middle unit, $V_2 = 1{\cdot}11 \, V_1 = 1{\cdot}11 \times 5{\cdot}52 = \mathbf{6.13\ kV}$

Voltage across bottom unit, $V_3 = 1{\cdot}342 \, V_1 = 1{\cdot}342 \times 5{\cdot}52 = \mathbf{7.4\ kV}$

(*ii*) String efficiency $$= \frac{\text{Voltage across string}}{\text{No. of insulators} \times V_3} \times 100 = \frac{19 \cdot 05}{3 \times 7 \cdot 4} \times 100 = \mathbf{85.8\%}$$

2.5.4- Methods of Improving String Efficiency:

- It has been seen above that potential distribution in a string of suspension insulators is not uniform.
- The maximum voltage appears across the insulator nearest to the line conductor and decreases progressively as the cross-arm is approached.

- It is necessary to equalize the potential across the various units of the string i.e., to improve the string efficiency.
- The various methods for this purpose are:
  1. By using longer cross-arms. The value of string efficiency depends upon the value of K i.e., ratio of shunt capacitance to mutual capacitance. The lesser the value of K, the greater is the string efficiency and more uniform is the voltage distribution. The value of K can be decreased by reducing the shunt capacitance. In order to reduce shunt capacitance, the distance of conductor from tower must be increased i.e., longer cross-arms should be used. However, limitations of cost and strength of tower do not allow the use of very long cross-arms. In practice, $K = 0 \cdot 1$ is the limit that can be achieved by this method.
  2. By grading the insulators. In this method, insulators of different dimensions are so chosen that each has a different capacitance. The insulators are capacitance graded i.e., they are assembled in the string in such a way that the top unit has the minimum capacitance, increasing progressively as the bottom unit (i.e., nearest to conductor) is reached. Since voltage is inversely proportional to capacitance, this method tends to equalize the potential distribution across the units in the string. This method has the disadvantage that a large number of different-sized

insulators are required. However, good results can be obtained by using standard insulators for most of the string and larger units for that near to the line conductor.

3. By using a guard ring. The potential across each unit in a string can be equalized by using a guard ring which is a metal ring electrically connected to the conductor and surrounding the bottom insulator as shown in the Fig. 2.13. The guard ring introduces capacitance between metal fittings and the line conductor. The guard ring is contoured in such a way that shunt capacitance currents $i1$, $i2$ etc. are equal to metal fitting line capacitance currents $i'1$, $i'2$ etc. The result is that same charging current I flows through each unit of string. Consequently, there will be uniform potential distribution across the units.

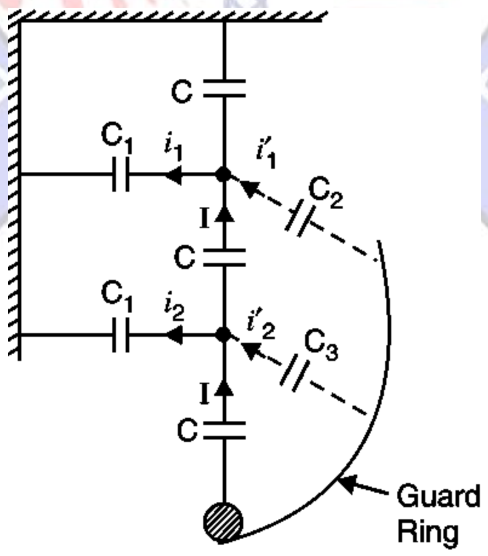


Fig. 2.13

While solving problems relating to string efficiency, the following points must be kept in mind:

(i) The maximum voltage appears across the disc nearest to the conductor (i.e., line conductor).

(ii) (ii) The voltage across the string is equal to phase voltage i.e., Voltage across string = Voltage between line and earth = Phase Voltage

(iii) (iii) Line Voltage = $\sqrt{3}$ × Voltage across string

Example 2.3:

Each line of a 3-phase system is suspended by a string of 3 indentical insulators of self-capacitance C farad. The shunt capacitance of connecting metal work of each insulator is 0·2 C to earth and 0·1 C to line. Calculate the string efficiency of the system if a guard ring increases the capacitance to the line of metal work of the lowest insulator to 0·3 C.

Solution. The capacitance between each unit and line is artificially increased by using a guard ring as shown in Fig. 8.21. This arrangement tends to equalise the potential across various units and hence leads to improved string efficiency. It is given that with the use of guard ring, capacitance of the insulator link-pin to the line of the lowest unit is increased from 0·1 C to 0·3 C.

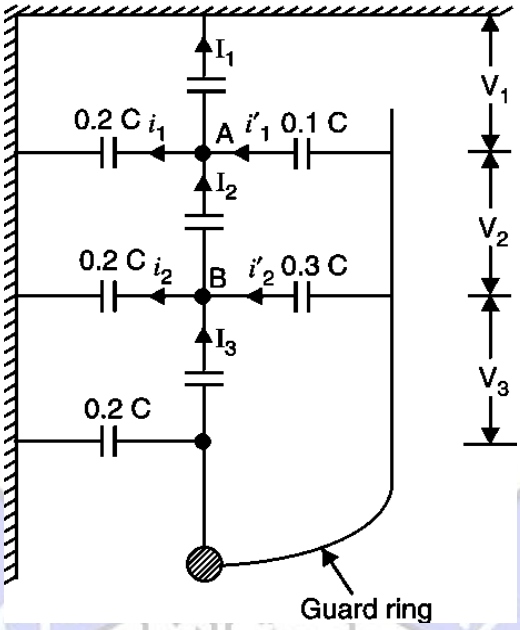


Fig. 8.21

At Junction A

$$I_2 + i'_1 = I_1 + i_1$$

or $$V_2\,\omega\,C + (V_2 + V_3)\,\omega \times 0{\cdot}1\,C = V_1\,\omega\,C + V_1 \times 0{\cdot}2\,C\,\omega$$

$$V_3 = 12\,V_1 - 11\,V_2 \qquad ...(i)$$

At Junction B

$$I_3 + i'_2 = I_2 + i_2$$

or $$V_3\,\omega\,C + V_3 \times 0{\cdot}3\,C \times \omega = V_2\,\omega\,C + (V_1 + V_2)\,\omega \times 0{\cdot}2\,C$$

or $$1{\cdot}3\,V_3 = 1{\cdot}2\,V_2 + 0{\cdot}2\,V_1 \qquad ...(ii)$$

Substituting the value of V_3 from exp. (*i*) into exp. (*ii*), we get,

$$1{\cdot}3\,(12\,V_1 - 11 V_2) = 1{\cdot}2\,V_2 + 0{\cdot}2\,V_1$$

or $$15{\cdot}5\,V_2 = 15{\cdot}4\,V_1$$

$\therefore$ $$V_2 = 15{\cdot}4\,V_1/15{\cdot}5 = 0{\cdot}993\,V_1 \qquad ...(iii)$$

Substituting the value of V_2 from exp. (*iii*) into exp. (*i*), we get,

$$V_3 = 12\ V_1 - 11 \times 0{\cdot}993\ V_1 = 1{\cdot}077\ V_1$$

Voltage between conductor and earth (*i.e.* phase voltage)

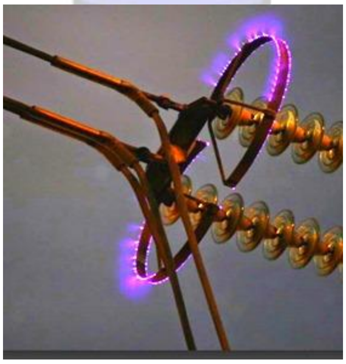
$$= V_1 + V_2 + V_3 = V_1 + 0{\cdot}993\ V_1 + 1{\cdot}077\ V_1 = 3{\cdot}07\ V_1$$

$$\text{String efficiency} = \frac{3\cdot 07\ V_1}{3 \times 1\cdot 077\ V_1} \times 100 = \mathbf{95\%}$$

2.6- Corona in Overhead Transmission Lines:

- When the applied voltage exceeds a certain value, called critical disruptive voltage, the conductors are surrounded by a faint violet glow called corona as shown below

- The phenomenon of corona is accompanied by a hissing sound, production of ozone, power loss and radio interference.
- The higher the voltage is raised, the larger and higher the luminous envelope becomes, and greater are the sound, the power loss and the radio noise.
- If the applied voltage is increased to breakdown value (about 30 kV per cm), a flash-over will occur between the conductors due to the breakdown of air insulation.



- The phenomenon of violet glow, hissing noise and production of ozone gas in an overhead transmission line is known as corona.

2.6.1- Factors Affecting Corona:

1- Atmosphere:

As corona is formed due to ionization of air surrounding the conductors. In the stormy weather, the number of ions is more than normal and corona occurs at much less voltage as compared with fair weather.

2- Conductor size:

The corona effect depends upon the shape and conditions of the conductors. Thus, a stranded conductor has irregular surface and hence gives rise to more corona that a solid conductor.

3- Spacing between conductors:

If the spacing between the conductors is made very large as compared to their diameters, there may not be any corona effect.

4- Line voltage:

The line voltage greatly affects corona. If it is low, there is no change in the condition of air surrounding the conductors and hence no corona is formed.

2.6.2- Advantages and Disadvantages of Corona:

Advantages:

- Due to corona formation, the air surrounding the conductor becomes conducting and hence virtual diameter of the conductor is increased. The increased diameter reduces the electrostatic stresses between the conductors.

Disadvantages:

- Corona is accompanied by a loss of energy. This affects the transmission efficiency of the line.
- Ozone is produced by corona and may cause corrosion of the conductor due to chemical action.
- The current drawn by the line due to corona is non-sinusoidal and hence non-sinusoidal voltage drop occurs in the line. This may cause inductive interference with neighboring communication lines.

2.6.3- Methods of Reducing Corona Effect:

1-By increasing conductor size:

- The voltage at which corona occurs is raised and hence corona effects are considerably reduced. This is one of the reasons that ACSR conductors which have a larger cross-sectional area are used in transmission lines.

2- By increasing conductor spacing:

By increasing the spacing between conductors, the voltage at which corona occurs is raised and hence corona effects can be eliminated. However, spacing cannot be increased too much otherwise the cost of supporting structure (e.g., bigger cross arms and supports) may increase to a considerable extent.

2.7- Sag in Overhead Transmission Lines:

- If the conductors are too much stretched between supports in a bid to save conductor material, the stress in the conductor may reach unsafe value and in certain cases the conductor may break due to excessive tension.
- In order to permit safe tension in the conductors, they are not fully stretched but are allowed to have a dip or sag as shown in fig. 2.14.

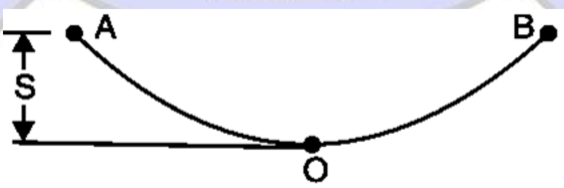


Fig. 2.14:

- The difference in level between points of supports and the lowest point on the conductor is called sag.

2.7.1- Calculation of Sag:

- In an overhead line, the sag should be adjusted that tension in the conductors is within safe limits.
- The tension is governed by conductor weight, effects of wind, ice loading and temperature variations.
- It is a standard practice to keep conductor tension less than 50% of its ultimate tensile strength.
- We now calculate sag and tension of a conductor when (1) supports are at equal levels and (2) supports are at unequal levels.

1-When supports are at equal levels:

- Consider a conductor between two equilevel supports A and B with O as the lowest point as shown in Fig. 2.15. It can be proved that lowest point will be at the mid-span.

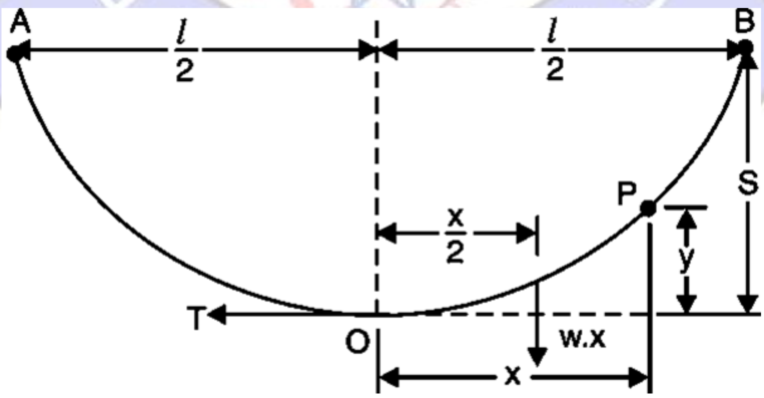


Fig. 2.15:

Let

l = Length of span

w = Weight per unit length of conductor

T = Tension in the conductor.

- Consider a point P on the conductor.
- Taking the lowest point O as the origin, let the co-ordinates of point P be x and y.
- Assuming that the curvature is so small that curved length is equal to its horizontal projection (i.e., OP = x), the two forces acting on the portion OP of the conductor are:

(a) The weight wx of conductor acting at a distance $x/2$ from O.
(b) The tension T acting at O.

Equating the moments of above two forces about point O, we get,

$$T y = w x \times \frac{x}{2}$$

or

$$y = \frac{w x^2}{2 T}$$

The maximum dip (sag) is represented by the value of y at either of the supports A and B.

At support A, $x = l/2$ and $y = S$

$$\therefore \quad \text{Sag,} \quad S = \frac{w(l/2)^2}{2T} = \frac{w\,l^2}{8\,T}$$

Example 2.4:

A 132 kV transmission line has the following data :

Wt. of conductor = 680 kg/km ; Length of span = 260 m

Ultimate strength = 3100 kg ; Safety factor = 2

Calculate the height above ground at which the conductor should be supported. Ground clearance required is 10 metres.

Solution.

Wt. of conductor/metre run, $w = 680/1000 = 0{\cdot}68$ kg

Working tension, $T = \dfrac{\text{Ultimate strength}}{\text{Safety factor}} = \dfrac{3100}{2} = 1550$ kg

Span length, $l = 260$ m

$$\therefore \quad \text{Sag} = \frac{w\,l^2}{8T} = \frac{0\cdot 68 \times (260)^2}{8 \times 1550} = 3{\cdot}7 \text{ m}$$

$\therefore$ Conductor should be supported at a height of $10 + 3{\cdot}7 =$ **13·7 m**

2-When supports are at unequal levels:

- In hilly areas, we generally come across conductors suspended between supports at unequal levels.
- Fig. 2.16 shows a conductor suspended between two supports A and B which are at different levels.

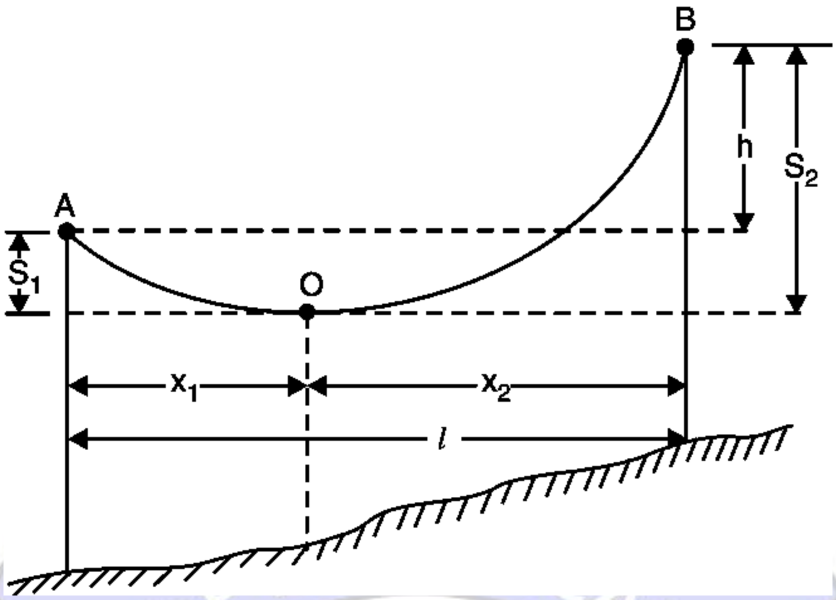


Fig. 2.16

- The lowest point on the conductor is O.

Let

l = Span length

h = Difference in levels between two supports

x_1 = Distance of support at lower level (*i.e., A*) from *O*

x_2 = Distance of support at higher level (*i.e. B*) from *O*

T = Tension in the conductor

If w is the weight per unit length of the conductor, then,

$$\text{Sag } S_1 = \frac{w\,x_1^{2*}}{2T}$$

and

$$\text{Sag } S_2 = \frac{w\,x_2^{2}}{2T}$$

Also

$$x_1 + x_2 = l \qquad \text{...(i)}$$

$$\text{Now} \quad S_2 - S_1 = \frac{w}{2T}[x_2^2 - x_1^2] = \frac{w}{2T}(x_2 + x_1)(x_2 - x_1)$$

$$\therefore \quad S_2 - S_1 = \frac{w\,l}{2T}(x_2 - x_1) \qquad [\because x_1 + x_2 = l]$$

$$\text{But} \quad S_2 - S_1 = h$$

$$\therefore \quad h = \frac{w\,l}{2T}(x_2 - x_1)$$

$$\text{or} \quad x_2 - x_1 = \frac{2\,Th}{w\,l} \qquad \text{...(ii)}$$

Solving exps. (*i*) and (*ii*), we get,

$$x_1 = \frac{l}{2} - \frac{T\,h}{w\,l}$$

$$x_2 = \frac{l}{2} + \frac{T\,h}{w\,l}$$

Having found x_1 and x_2, values of S_1 and S_2 can be easily calculated.

Example 2.5:

The towers of height 30 m and 90 m respectively support a transmission line conductor at water crossing. The horizontal distance betwen the towers is 500 m. If the tension in the conductor is 1600 kg, find the minimum clearance of the conductor and water and clearance mid-way between the supports. Weight of conductor is 1·5 kg/m. Bases of the towers can be considered to be at water level.

Solution. Fig. 8.28 shows the conductor suspended between two supports *A* and *B* at different levels with *O* as the lowest point on the conductor.

Here, $l = 500$ m ; $w = 1{\cdot}5$ kg ; $T = 1600$ kg.

Difference in levels between supports, $h = 90 - 30 = 60$ m. Let the lowest point *O* of the conductor be at a distance x_1 from the support at lower level (*i.e.*, support *A*) and at a distance x_2 from the support at higher level (*i.e.*, support *B*).

Obviously, $\quad x_1 + x_2 = 500$ m $\qquad$ ...(*i*)

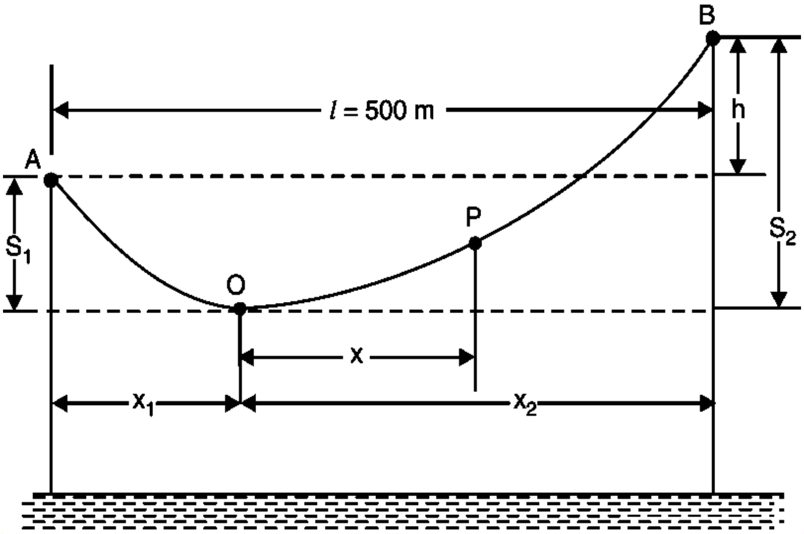


Fig. 8.28

Now Sag $S_1 = \frac{w\,x_1^2}{2T}$ and Sag $S_2 = \frac{w\,x_2^2}{2T}$

$$\therefore \quad h = S_2 - S_1 = \frac{w\,x_2^2}{2T} - \frac{w\,x_1^2}{2T}$$

or $$60 = \frac{w}{2T}(x_2 + x_1)(x_2 - x_1)$$

$$\therefore \quad x_2 - x_1 = \frac{60 \times 2 \times 1600}{1{\cdot}5 \times 500} = 256 \text{ m} \qquad ...(ii)$$

Solving exps. (*i*) and (*ii*), we get, $x_1 = 122$ m; $x_2 = 378$ m

Now, $$S_1 = \frac{w\,x_1^2}{2T} = \frac{1{\cdot}5 \times (122)^2}{2 \times 1600} = 7 \text{ m}$$

Clearance of the lowest point *O* from water level

$$= 30 - 7 = \mathbf{23\ m}$$

Let the mid-point *P* be at a distance *x* from the lowest point *O*.

Clearly, $$x = 250 - x_1 = 250 - 122 = 128 \text{ m}$$

Sag at mid-point *P*, $$S_{mid} = \frac{w\,x^2}{2T} = \frac{1{\cdot}5 \times (128)^2}{2 \times 1600} = 7{\cdot}68 \text{ m}$$

2.7.2- Effect of wind and ice loading:

- The above formula for sag is true only in still air and at normal temperature when the conductor is acted by its weight only.
- However, in actual practice, a conductor may have ice coating and simultaneously subjected to wind pressure.
- The weight of ice acts vertically downwards i.e., in the same direction as the weight of conductor.
- The force due to the wind is assumed to act horizontally i.e., at right angle to the projected surface of the conductor.
- Hence, the total force on the conductor is the vector sum of horizontal and vertical forces as shown in Fig. 2.17 (iii).

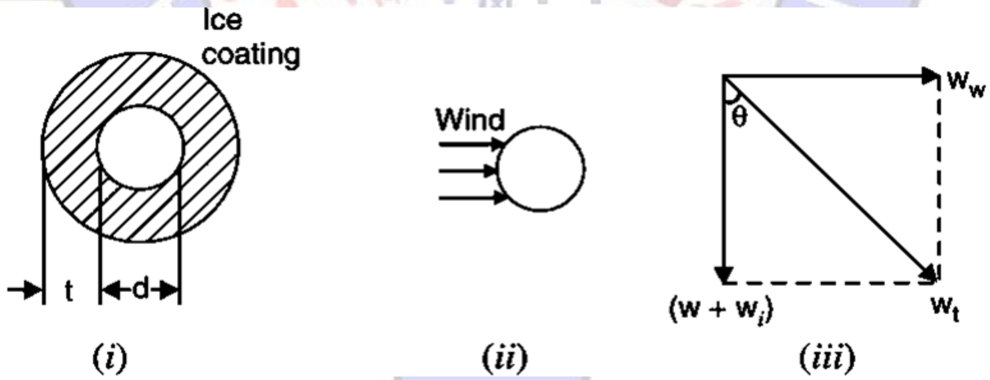


Fig. 2.17:

Total weight of conductor per unit length is

$$w_t = \sqrt{(w + w_i)^2 + (w_w)^2}$$

where w = weight of conductor per unit length

w_i = weight of ice per unit length

= density of ice × $\pi\, t\,(d + t)$*

w_w = wind force per unit length

= wind pressure × $[(d + 2t) \times 1]$

When the conductor has wind and ice loading also, the following points may be noted :

(i) The conductor sets itself in a plane at an angle θ to the vertical where

$$\tan\theta = \frac{w_w}{w + w_i}$$

(ii) The sag in the conductor is given by :

$$S = \frac{w_t\, l^2}{2T}$$

Hence S represents the slant sag in a direction making an angle θ to the vertical.

(iii) The vertical sag = $S \cos\theta$

Example 2.6:

A transmission line has a span of 150 m between level supports. The tension in the conductor is 2000 kg. If the wind pressure is 1·5 kg/m length, calculate the sag. What is the vertical sag? . Wt. of conductor/m length, $w = 1{\cdot}98$ kg

Solution.

Span length, $l = 150$ m; Working tension, $T = 2000$ kg

Wind force/m length of conductor. $w_w = 1{\cdot}5$ kg

Wt. of conductor/m length, $w = 1{\cdot}98$ kg

Total weight of 1 m length of conductor is

$$w_t = \sqrt{w^2 + w_w^2} = \sqrt{(1\cdot98)^2 + (1\cdot5)^2} = 2{\cdot}48 \text{ kg}$$

$$\therefore \quad \text{Sag,} \quad S = \frac{w_t\, l^2}{8T} = \frac{2\cdot48 \times (150)^2}{8 \times 2000} = \mathbf{3{\cdot}48\ m}$$

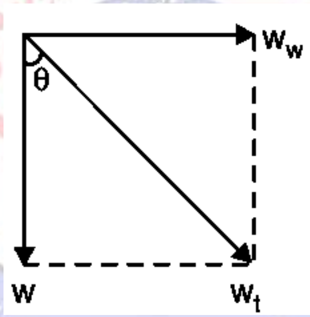
This is the value of slant sag in a direction making an angle θ with the vertical. Referring to Fig. 8.27, the value of θ is given by ;

$$\tan\theta = w_w/w = 1{\cdot}5/1{\cdot}98 = 0{\cdot}76$$

$$\therefore \quad \theta = \tan^{-1} 0{\cdot}76 = 37{\cdot}23^\circ$$

$$\therefore \quad \text{Vertical sag} = S\cos\theta$$

$$= 3{\cdot}48 \times \cos 37{\cdot}23^\circ = \mathbf{2{\cdot}77\ m}$$



Chapter Three

Underground Cables

3.1- Introduction
3.2- Underground Cables
3.3- Construction of Cables
3.4- Insulating Materials for Cables
3.5- Laying of Underground Cables
3.6- Types of Cable Faults

3.1- Introduction:

- Electric power can be transmitted or distributed either by overhead system or by underground cables.
- The underground cables have several advantages such as low maintenance cost, smaller voltage drop and better general appearance.
- However, their major drawback is the higher installation cost compared with the overhead system.
- For this reason, underground cables are employed where it is impracticable to use overhead lines.
- Such locations may be populated areas or where maintenance conditions do not permit the use of overhead construction.

3.2- Underground Cables:

- An underground cable essentially consists of one or more conductors covered with suitable insulation and surrounded by a protecting cover.
- The type of cable to be used will depend upon the working voltage and service requirements. In general, a cable must fulfil the following necessary requirements:

1. The conductor used in cables should be stranded copper or aluminium of high conductivity.
2. The conductor size should be able to carry the desired load current without overheating and causes voltage drop.
3. The cable must have proper thickness of insulation in order to give high degree of safety.

3.3- Construction of Cables:

- Fig. 3.1 shows the general construction of a 3-conductor cable. The various parts are:

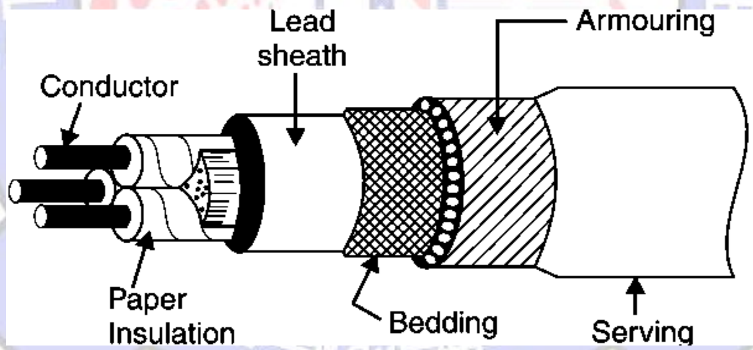


Fig. 3.1:

1. **Cores or Conductors:** A cable may have one or more than one core (conductor) depending upon the type of service for which it is intended. For instance, the 3-conductor cable shown in Fig. 3.1 is used for 3-phase service.

2. **Insulation:** Each core or conductor is provided with a suitable thickness of insulation, the thickness of layer depending upon the voltage to be withstood by the cable. The impregnated paper is one of the commonly materials used for insulation.
3. **Metallic sheath:** In order to protect the cable from moisture, gases or other damaging liquids in the soil.
4. **Bedding:** The purpose of bedding is to protect the metallic sheath against corrosion and from mechanical injury due to armouring.
5. **Armouring:** It is used to protect the cable from mechanical injury during the course of handling. Armouring may not be done in the case of some cables.
6. **Serving:** In order to protect armouring from atmospheric conditions.

3.4- Insulating Materials for Cables:

- The satisfactory operation of a cable depends upon the characteristics of insulation used. In general, the insulating material used in cables should have the following properties:

1. High insulation resistance to avoid leakage current.
2. High dielectric strength to avoid electrical breakdown of the cable.
3. High mechanical strength to withstand the mechanical handling of cables.
4. Low cost.

3.5- Laying of Underground Cables:

- The reliability of underground cable network depends upon the proper laying and attachment of fittings i.e., cable end boxes, joints, branch connectors etc.
- There are three main methods of laying underground cables:

  1-Direct laying: Fig. 3.2 shows the direct method of laying underground cables which is simple and cheap.

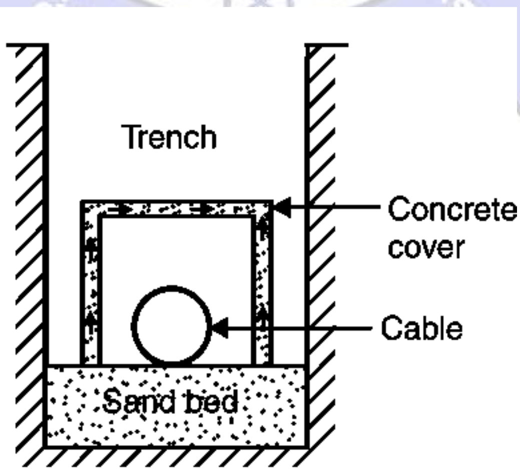


Fig. 3.2

- In this method, a trench of about 1.5 meters deep and 45 cm wide.
- The trench is covered with a layer of fine sand (of about 10 cm thickness) and the cable is laid over this sand bed.
- The sand prevents the entry of moisture from the ground.
- After the cable has been laid in the trench, it is covered with another layer of sand of about 10 cm thickness.
- The trench is then covered with concrete cover in order to protect the cable from mechanical injury.

Advantages:

- It is a simple and less costly method.
- It is a clean and safe method as the cable is invisible from external disturbances.

Disadvantages:

- The alterations in the cable network cannot be made easily.
- The maintenance cost is very high.
- Localization of fault is difficult.
- It cannot be used in crowded areas where excavation is expensive.

2-Draw-in system: In this method as shown in Fig. 3.3 shows, duct of concrete is laid in the ground with manholes at suitable positions along the cable route.

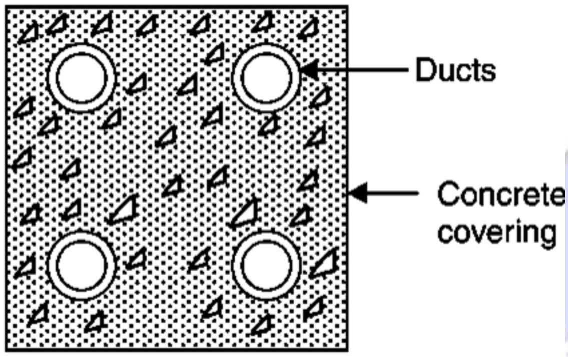


Fig. 3.3

- The cables to be laid in this way need not be armoured but must be provided with serving in order to protect them when being pulled into the ducts.

Advantages:

- Repairs, alterations or additions to the cable network can be made without opening the ground.
- It can be used in crowded areas where excavation is expensive.

Disadvantages:

- The initial cost is very high.

This method is generally used for short length cable routes such as in workshops or road crossings.

3-Solid system: In this method of laying, the cable is laid in open pipes in earth along the cable route. After the cable is laid in position, the troughing is filled with a asphaltic compound and covered over.

Disadvantages:

- It is more expensive than direct laid system.

3.6- Types of Cable Faults:

- Cables are generally laid directly in the ground or in ducts in the underground distribution system.
- For this reason, there are little chances of faults in underground cables.
- However, if a fault does occur, it is difficult to locate and repair the fault because conductors are not visible.
- So, the following are the faults most likely to occur in underground cables:

1. Open-circuit fault:

- When there is a break in the conductor of a cable, it is called open-circuit fault.
- The open-circuit fault can be checked by a megger.

- For this purpose, the three conductors of the 3-core cable at the far end are shorted and earthed.
- Then resistance between each conductor and earth is measured by a megger.
- The megger will indicate zero resistance in the circuit of the conductor that is not broken.
- However, if the conductor is broken, the megger will indicate infinite resistance in its circuit.

2. Short-circuit fault:

- When two conductors of a multi-core cable come in electrical contact with each other due to insulation failure, it is called a short-circuit fault.
- Again, we can use megger to check this fault.
- For this purpose, the two terminals of the megger are connected to any two conductors.
- If the megger gives zero reading, it indicates short-circuit fault between these conductors.
- The same step is repeated for other conductors taking two at a time.

3. Earth fault:

- When the conductor of a cable comes in contact with earth, it is called earth fault or ground fault.
- To identify this fault, one terminal of the megger is connected to the conductor and the other terminal connected to earth.
- If the megger indicates zero reading, it means the conductor is earthed.
- The same procedure is repeated for other conductors of the cable.

Chapter Four

Economic Operation of Power System

4.1- Introduction

4.2- Formulation of Economic Dispatch Problem

4.3- Classical Economic Dispatch without line losses

4.4- Generating Limits

4.5- Classical Economic Dispatch with line losses

4.6- Examples

4.1- Introduction

- The Economic operation is very important for a power system to return a profit on the capital invested.
- The operation economics can be divided into two parts.
    1. **Problem of economic dispatch**, which deals with determining the power output of each plant to meet the specified load, such that the overall fuel cost is minimized.
    2. **Problem of optimal power flow**, which deals with minimum–loss delivery to minimize losses in the system.
- The factors influencing the cost of generation are the generator efficiency, fuel cost, and transmission losses.
- The most efficient generator may not give minimum cost, since it may be located in a place where fuel cost is high.
- Further, if the plant is located far from the load centers, transmission losses may be high and running the plant may become uneconomical.
- The economic dispatch problem basically determines the generation of different plants to minimize total operating cost.
- Power generating plants like nuclear plants, thermal plants, and diesel plants, may require capital investment of millions of rupees.
- The economic dispatch is however determined in terms of fuel cost per unit power generated and does not include capital investment, maintenance, depreciation, start-up and shut down costs etc.

- The main aim in the economic dispatch problem is to minimize the total cost of generating real power (production cost) at various stations.

4.2- Formulation of Economic Dispatch Problem:

- A typical input-output curve which is a plot of fuel input for a fossil-fuel plant per hour versus power output of the unit in megawatts is shown in fig. 4.1. The major component of generator operating cost is the fuel input/hour

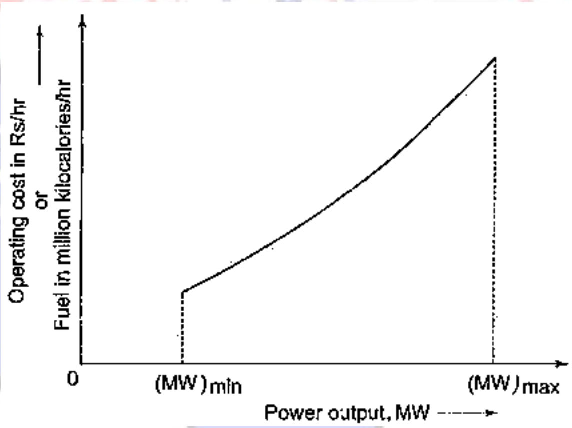


Fig. 4.1

- A typical plot of incremental fuel cost versus power output is shown in Fig. 4.2.

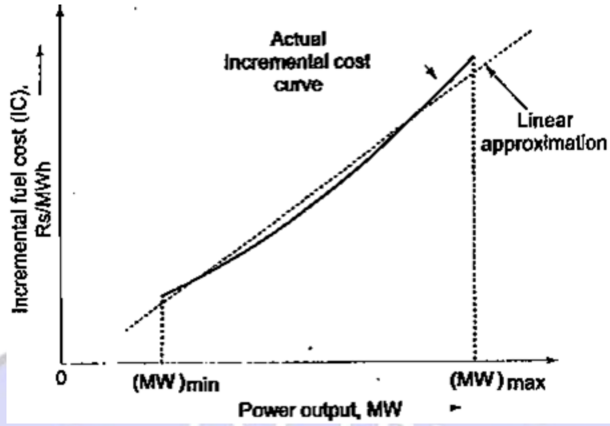


Fig. 4.2

The unit of the incremental fuel cost is Rs / MWh or $ /MWh.

In general, the fuel cost F_i for a plant, is approximated as a quadratic function of the generated output P_{Gi}.

$$F_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \text{ Rs/h}$$

The incremental fuel cost is given by

$$\frac{dF_i}{dP_{Gi}} = b_i + 2c_i P_{Gi} \quad \text{Rs/MWh}$$

- The incremental fuel cost is a measure of how costly it will be producing an increment of power.

- Figure 4.3 shows the configuration that will be studied in this section. This system consists of N thermal-generating units connected to a single bus-bar serving a received electrical load P_{load}.

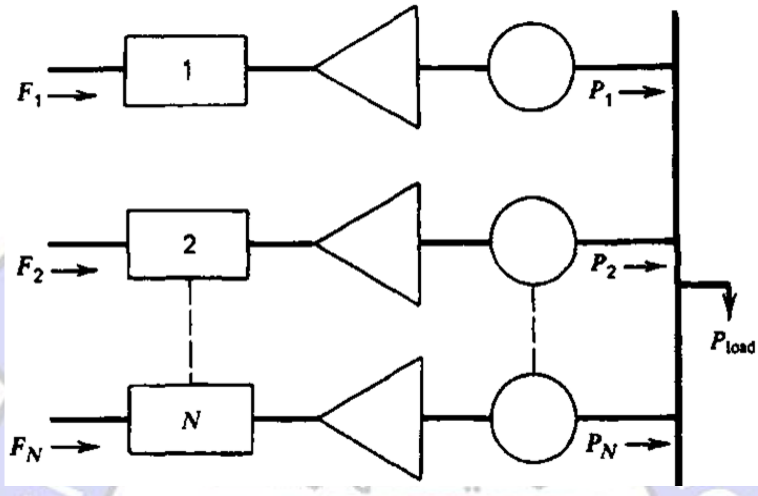


Fig. 4.3:

- Mathematically speaking;
- The input to each unit F_i , represents the cost rate of the unit.
- The output of each unit P_i, is the electrical power generated by that particular unit.
- The total cost rate of this system is the sum of the costs of each of the individual units.

$$F_T = F_1 + F_2 + F_3 + \cdots + F_N$$
$$= \sum_{i=1}^{N} F_i(P_i) \qquad (3.1)$$

4.3- Classical Economic Dispatch without line losses:

- The simplest case of economic dispatch is the case when transmission losses are neglected.
- The model does not consider the system configuration or line impedances. Since losses are neglected, the total generation is equal to the total demand P_D .
- Mathematically, the problem is to minimize F_T subject to the constraint that the sum of the powers generated must equal the received load as follows:

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

Where F_T= total cost

P_{Gi}= generation of plant i

P_D= total demand

- This is a constrained optimization problem, which can be solved by Lagrange's Method.

- The problem is restated below:

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$P_D - \sum_{i=1}^{n_g} P_{Gi} = 0$$

- The augmented cost function is given by

$$L = F_T + \lambda(P_D - \sum_{i=1}^{n_g} P_{Gi})$$

- The minimum is obtained when

$$\frac{\partial L}{\partial P_{Gi}} = 0$$

$$\frac{\partial L}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda = 0$$

- The second equation is simply the original constraint of the problem. The cost of a plant F_i depends only on its own output P_{Gi} , hence

$$\frac{\partial F_T}{\partial P_{Gi}} = \frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$$

Using the above,

$$\frac{\partial L}{dP_{Gi}} = \frac{dF_i}{dP_{Gi}} - \lambda = 0 \qquad i = 1, 2, \text{-----------}, n_g$$

We can write $$b_i + 2c_i P_{Gi} = \lambda \qquad i = 1, 2, \text{-----------}, n_g$$

- Simply stated, for economic generation scheduling to meet a particular load demand, when transmission losses are neglected, all plants must operate at equal incremental production costs, subject to the constraint that the total generation be equal to the demand.

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

- These conditions and inequalities may be summarized as shown in the set of equations making up Eq. 3.5.

Example 4.1:

2 units, $P_D = 700$ MW,

$F_1 = 10P_1 + 8\times10^{-3} P_1^2$ (\$/hr)

$F_2 = 8P_2 + 9\times10^{-3} P_2^2$ (\$/hr)

(1) Find P_1 and P_2 (output power of each station (unit))

(2) Incremental fuel cost

(3) Total fuel cost.

Sol.:

$$\frac{dF_i}{dP_i} = \lambda \quad \text{where, } i = 1, 2$$

عدد الوحدات

$$\frac{dF_1}{dP_1} = \lambda \longrightarrow (1)$$

$$= 10 + 16 \times 10^{-3} P_1 = \lambda$$

$$\therefore P_1 = \frac{\lambda - 10}{16 \times 10^{-3}}$$

$$\therefore P_1 = 62.5\lambda - 625 \longrightarrow (1)$$

$$\frac{dF_2}{dP_2} = 8 + 18 \times 10^{-3} P_2 = \lambda \longrightarrow (2)$$

$$P_2 = \frac{\lambda - 8}{18 \times 10^{-3}}$$

$$P_2 = 55.6\lambda - 444.4 \longrightarrow (2)$$

$$P_D = P_1 + P_2$$

$$P_1 + P_2 = 700 \longrightarrow (3)$$

تعويض معادلة (1) (2) في معادلة (3)

$$(62.5\lambda - 625) + (55.6\lambda - 444.4) = 700$$

$$118.1\lambda - 1069.4 - 700 = 0$$

$$118.1\lambda = 1769.4$$

$$\therefore \lambda = \frac{1769.4}{118.1} \Rightarrow \lambda = 14.98 \ \$/MWhr$$

نعوض قيمة λ في معادلة رقم (1)

$$P_1 = 62.5(14.98) - 625$$

$$= 936.3887 - 625$$

$$= 311.3887 \ MW$$

$$P_2 = 55.6\lambda - 444.4$$

$$= 55.6(14.98) - 444.4$$

$$= 832.888 - 444.4$$

$$= 388.488 \ MW$$

$F_{total} = F_1 \Big|_{P_1 = 311.388} + F_2 \Big|_{P_2 = 388.488}$

where,

$F_1 = 10 P_1 + 8 \times 10^{-3} P_1^2$

$= 10(311.388) + 8 \times 10^{-3} (311.388)^2$

$F_1 = 3113.88 + 775.6999$

$F_1 = 3889.58$

$F_2 = 8 P_2 + 9 \times 10^{-3} P_2^2$

$= 8(388.488) + 9 \times 10^{-3} (388.488)^2$

$= 3107.904 + 1358.306$

$= 4466.21$

$\therefore F_T = 8355.79 \ (\$/hr)$

4.4- Generating Limits:

- It is not always necessary that all the units of a plant are available to share a load.
- Some of the units may be taken off due to scheduled maintenance. Also, it is not necessary that the less efficient units are switched off during off peak hours.
- There is a certain amount of shut down and startup costs associated with shutting down a unit during the off-peak hours and servicing it back on-line during the peak hours.
- The optimal load dispatch problem must then incorporate this startup and shut down cost for without endangering the system security.
- The power generation limit of each unit is then given by the inequality constraints;

$$P_{\min,i} \le P_i \le P_{\max,i}, \quad i = 1, \ldots, N$$

The maximum limit P_{max} is the upper limit of power generation capacity of each unit. On the other hand, the P_{min} is the lower limit

An operational unit must produce a minimum amount of power

Example 4.2:

The fuel costs of three generating units are given by;

$$f_1 = \frac{0.8}{2} P_1^2 + 10P_1 + 25 \text{ Rs./h}$$

$$f_2 = \frac{0.7}{2} P_2^2 + 5P_2 + 20 \text{ Rs./h}$$

$$f_3 = \frac{0.95}{2} P_3^2 + 15P_3 + 35 \text{ Rs./h}$$

The generation limits of the units are;

$$30\,\text{MW} \le P_1 \le 500\,\text{MW}$$

$$30\,\text{MW} \le P_2 \le 500\,\text{MW}$$

$$30\,\text{MW} \le P_3 \le 250\,\text{MW}$$

The total load that these units supply varies between 90 MW and 1250 MW. Assuming that all the three units are operational all the time. Compute the economic operating settings as the load changes.

Sol.:

The incremental costs of these units are

$$\frac{df_1}{dP_1} = 0.8P_1 + 10 \text{ Rs./MWh}$$

$$\frac{df_2}{dP_2} = 0.7P_2 + 5 \text{ Rs./MWh}$$

$$\frac{df_3}{dP_3} = 0.95P_3 + 15 \text{ Rs./MWh}$$

At the minimum load the incremental cost of the units are

$$\frac{df_1}{dP_1} = \frac{0.8}{2}30^2 + 10 = 34 \text{ Rs./MWh}$$

$$\frac{df_2}{dP_2} = \frac{0.7}{2}30^2 + 5 = 26 \text{ Rs./MWh}$$

$$\frac{df_3}{dP_3} = \frac{0.95}{2}30^2 + 15 = 43.5 \text{ Rs./MWh}$$

- Since units 1 and 3 have higher incremental cost, they must therefore operate at 30 MW each.
- The incremental cost during this time will be due to unit-2 and will be equal to 26 Rs. /MWh.
- With the generation of units 1 and 3 remaining constant, the generation of unit-2 is increased till its incremental cost is equal to that of unit-1, i.e., 34 Rs. /MWh.

$$\frac{df_2}{dP_2} = 0.7P_2 + 5 = 34 \text{ Rs./MWh}$$

- This is achieved when P_2 is equal to 41.4286 MW, at a total power of 101.4286 MW.

- An increase in the total load beyond 101.4286 MW is shared between units 1 and 2, till their incremental costs are equal to that of unit-3, i.e., 43.5 Rs. /MWh.

$$\frac{df_1}{dP_1} = 0.8P_1 + 10 = 43.5 \text{ Rs./MWh}$$

$$\frac{df_2}{dP_2} = 0.7P_2 + 5 = 43.5 \text{ Rs./MWh}$$

- This point is reached when P_1 = 41.875 MW and P_2 = 55 MW.
- The total load that can be supplied at that point is equal to 126.875.
- From this point onwards the load is shared between the three units in such a way that the incremental costs of all the units are same.
- For example, for a total load of 200 MW, we have;

$$P_1 + P_2 + P_3 = 200$$
$$0.8P_1 + 10 = 0.7P_2 + 5$$
$$0.7P_2 + 5 = 0.95P_3 + 15$$

- Solving the above three equations we get P_1 = 66.37 MW, P_2 = 80 MW and P_3 = 50.63 MW and an incremental cost (ʎ) of 63.1 Rs. /MWh.
- In a similar way the economic dispatch for various other load settings is computed.
- The load distribution and the incremental costs are listed in Table 5.1 for various total power conditions.

Table 5.1 Load distribution and incremental cost for the units

P_T (MW)	P_1 (MW)	P_2 (MW)	P_3 (MW)	λ (Rs./MWh)
90	30	30	30	26
101.4286	30	41.4286	30	34
120	38.67	51.33	30	40.93
126.875	41.875	55	30	43.5
150	49.62	63.85	36.53	49.7
200	66.37	83	50.63	63.1
300	99.87	121.28	78.85	89.9
400	133.38	159.57	107.05	116.7
500	166.88	197.86	135.26	143.5
600	200.38	236.15	163.47	170.3
700	233.88	274.43	191.69	197.1
800	267.38	312.72	219.9	223.9
906.6964	303.125	353.5714	250	252.5
1000	346.67	403.33	250	287.33
1100	393.33	456.67	250	324.67
1181.25	431.25	500	250	355
1200	450	500	250	370
1250	500	500	250	410

- At a total load of 906.6964, unit-3 reaches its maximum load of 250 MW.
- From this point onwards then, the generation of this unit is kept fixed and the economic dispatch problem involves the other two units.
- For example, for a total load of 1000 MW, we get the following two equations;

$$P_1 + P_2 = 1000 - 250$$
$$0.8P_1 + 10 = 0.7P_2 + 5$$

Solving which we get P_1 = 346.67 MW and P_2 = 403.33 MW and an incremental cost of 287.33 Rs./MWh.

- Furthermore, unit-2 reaches its peak output at a total load of 1181.25.
- Therefore, any further increase in the total load must be supplied by unit-1 and the incremental cost will only be borne by this unit.
- The power distribution curve is shown in Fig. 5.1.

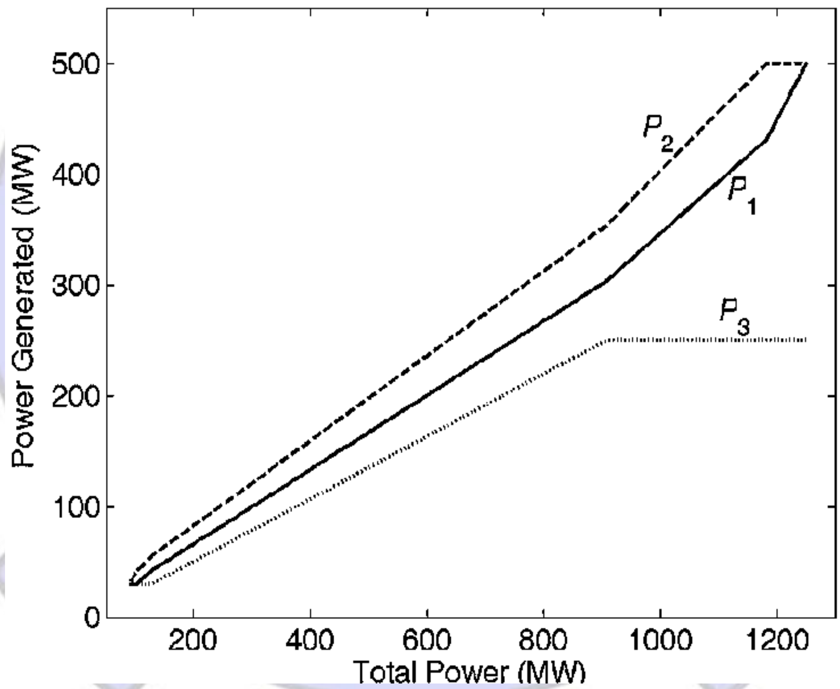


Fig. 4.4:

4.5- Classical Economic Dispatch with line losses:

- When transmission distances are large, the transmission losses are a significant part of the generation and have to be considered in the generation schedule for economic operation.
- The mathematical formulation is now stated as;

Minimize $F_T = \sum_{i=1}^{n_g} F_i$

Such that $\sum_{i=1}^{n_g} P_{Gi} = P_D + P_L$

Where P_L is the total loss

- The Lagrange function is now written as;

$$L = F_T + \lambda (P_D - \sum_{i=1}^{n_g} P_{Gi} - P_L)$$

- The minimum point is obtained when

$$\frac{\partial L}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda (1 - \frac{\partial P_L}{\partial P_{Gi}}) = 0 \qquad i=1,2,..........n_g$$

$$\frac{\partial L}{\partial \lambda} = P_D - \sum_{i=1}^{n_g} P_{Gi} - P_L = 0 \qquad \text{(same as the constraint)}$$

Since $$\frac{\partial F_T}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$$

$$\frac{dF_i}{dP_{Gi}} + \lambda \frac{dP_L}{dP_{Gi}} = \lambda$$

$$\lambda = \frac{dF_i}{dP_{Gi}} \left(\frac{1}{1 - \frac{dP_L}{dP_{Gi}}} \right)$$

The term $\frac{1}{1 - \frac{dP_L}{dP_{Gi}}}$ is called the penalty factor of plant i, L_i.

The coordination equations including losses are given by

$$\lambda = \frac{dF_i}{dP_{Gi}} L_i \qquad i = 1, 2, \ldots\ldots\ldots\ldots, n_g$$

- The minimum operation cost is obtained when the product of the incremental fuel cost and the penalty factor of all units is the same, when losses are considered.

Example 4.3:

The fuel costs of two power plants are;

$$f_1 = \frac{0.8}{2} P_1^2 + 10P_1 + 25 \text{ Rs./h}$$

$$f_2 = \frac{0.7}{2} P_2^2 + 6P_2 + 20 \text{ Rs./h}$$

It is assumed that the transmission loss is defined in terms of the two units as;

$$P_{LOSS} = 0.9\times10^{-4} P_1^2 + 1.4\times10^{-5} P_1 P_2 + 0.8\times10^{-4} \times P_2^2 \text{ MW}$$

Find the output powers of the two plants (P_1 and P_2) and the total power supplied to the load. Let us assume that the incremental cost is;

$$\lambda = 150 \text{ Rs./MWh}$$

Sol.:

Consider

$$L_i = \frac{1}{1 - \frac{dP_L}{dP_{Gi}}}$$

Then, we have

$$L_1 = \frac{1}{1-1.8\times10^{-4}P_1-1.4\times10^{-5}P_2}$$

and

$$L_2 = \frac{1}{1-1.4\times10^{-5}P_1-1.6\times10^{-4}P_2}$$

$$150 = \frac{0.8P_1+10}{1-1.8\times10^{-4}P_1-1.4\times10^{-5}P_2}$$

$$150 = \frac{0.7P_2+6}{1-1.4\times10^{-5}P_1-1.6\times10^{-4}P_2}$$

Rearranging the above two equations we get

$$\begin{bmatrix}0.8270 & 0.0021\\ 0.0021 & 0.7240\end{bmatrix}\begin{bmatrix}P_1\\ P_2\end{bmatrix}=\begin{bmatrix}140\\ 144\end{bmatrix}$$

The solution of the above equation produces

$P_1 = 168.78$ MW and $P_2 = 198.41$ MW.

The total power loss is then

$$P_{LOSS} = 0.9\times10^{-4}\times(168.78)^2+1.4\times10^{-5}\times(168.78)\times(198.41)+0.8\times10^{-4}\times(198.41)^2$$

$$= 6.18 \text{ MW}$$

Therefore the total power supplied to the load is

$$P_T = P_1+P_2-P_{LOSS} = 168.78+198.41-6.18 = 361 \text{ MW}$$